

Pedagogical “Toy” Model of Monetary Policies

Reference: <http://www2.physics.umd.edu/~yakovenk/econophysics>

We will examine the results of different monetary policies on a small economic system (a city of 50,000 inhabitants). We will use a pedagogical model to investigate policies. A model simple enough to be tractable using a personal computer running some math software, but having some elements of complexity. The basic assumption of our model is that the behavior of each individual is influenced by so many factors that the net result of their transactions is random.

This Model reveals that random processes alone will create disparities of wealth. However, our concept of Social Justice requires creating some means for caring for the less fortunate (fortunate: favored by or involving good luck or fortune; lucky). Other important factors that must be balanced with Social Justice are market efficiency and provision of capital for economic growth, and technical innovation. We will study efficiency using Thermodynamics.

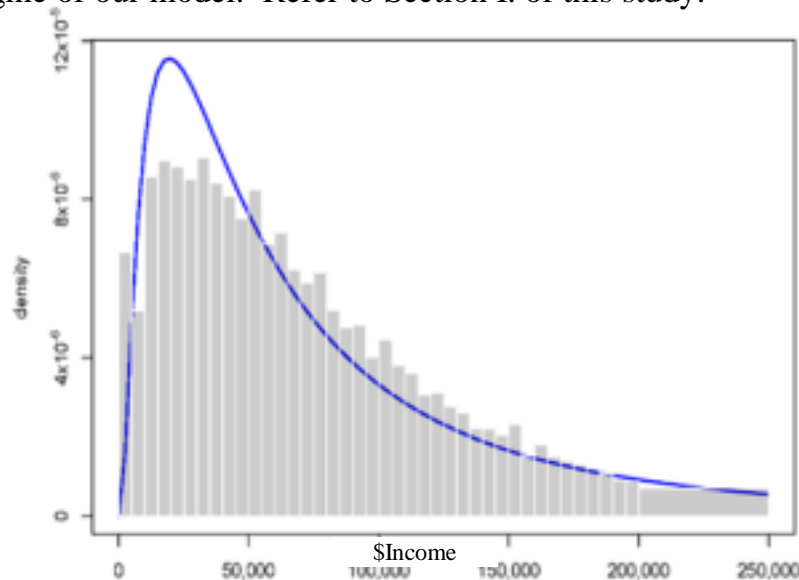
Greatest Social Good: Wealth Equity Considerations

Our Goal is to model an economic structure that achieves the “Greatest Social Good.” However, the Greatest Social Good is **subjective**. There are different notions of what is equitable.

We will model the structure of several different Economic Policies by varying the microeconomic parameters. In our models, no resource is ever allocated to make one individual better-off without making another individual worse-off. Neoclassical economists would view this type of exchange as “self-evidently rational.” This concept is called Pareto-optimal. According to Neoclassical thinking, the Pareto-optimal concept maximizes economic utility, but this does not result in a socially equitable distribution.

In microeconomic theory, utility is used as a measure of customer satisfaction in the exchange of goods and services.

While economists’ models traditionally regard humans as rational beings who always make intelligent decisions, econophysicists argue that in large economic systems the behavior of each individual is influenced by so many factors that the **net result is random**. Below is a histogram of Family Income Distribution in the US. This kind of curve is also generated by the statistical engine of our model. Refer to Section I. of this study.



We will look at the results of the microeconomic Pareto Efficient Economy. We will model the dynamics of this economy using a statistical methodology: The result of the exchanges will be randomly determined. One can think of these exchanges as betting or gambling, that is, the outcome is determined by chance alone. We will model the wealth of our economy by using a statistical engine that generates random sets of economic exchanges.

In models other than the Pareto, we will use Taxes and Subsidies to effect a “fairer” distribution of resources, that is, with the intent of maximizing the Social Good. We will define the state of maximum Social Good, as one that minimizes the number of people in “poverty.”

“Poverty” is a subjective concept. Let us arbitrarily define poverty as having a wealth less than \$15,000. For each economic policy, we can measure the percentage of people in poverty, that is, wealth less than \$15,000.

In some models the total of all economic resources is constant, that is, no monetary inflation. In others, subsidies to the agents exceeds collected income taxes. This results in inflation of currency.

We will model the Monetary Policy (e.g. Tax Redistribution/Equity) of a city (call it Econoville) of 50,000 people (economic agents). The model starts where everyone has the same amount of money or wealth of \$100,000. Financial transactions are done by randomly selecting a pair of agents. For every transaction, they toss a coin, one person gains a random fraction of money and the other loses an equal amount. The amount of the transaction is determined by a random fraction of the “average wealth” of the agents.” The analysis was done using two different types of “averages.” One “average” was the average amount of all the agents. The second “average” was the average only of the two agents in the transaction. The results using the two different averages gave distributions that had roughly the same shape and peak number of people for a given wealth. The calculations used in this analysis were based on the average wealth of the two agents. Using this type of average gives a faster convergence to the equilibrium values. Transactions are not allowed if the “bet” amount is less than the wealth of either one of the agents. No one is allowed to go into debt.

There are four parameters that we will independently vary in this economy.

1. The total number of transactions, N , between random pairs of the 50,000 agents.
2. Tax Rate
3. Types of Taxes: Flat, Marginal, Progressive
4. Frequency of Tax Collection,
5. Amount of Subsidies
6. Rules for the Distribution of Subsidies across the population.

At the end of the Number of Transactions (N) we will examine the results on four factors: Distribution of Wealth, the values of minimum and maximum wealth, and the percent of inflation.

We will examine the statistical effects of Five Different Monetary Policy Models:

A Pareto distribution is a skewed, heavy-tailed distribution that is sometimes used to model that distribution of incomes. The basis of the distribution is that a high proportion of a population has low income while only a few people have very high incomes.

The Pareto Principle (or 80-20 rule) states that 80% of income accrues to the top 20% of income recipients. Example: 20% of a Sales Force makes 80% of the sales.

The Pareto distribution applies to many different areas. It is used in Actuarial Analysis to determine risks for deaths in rare circumstances.

The Pareto distribution is very similar to a Lognormal Distribution, which is also used often to model the distribution of incomes. The Pareto is usually a better match to the upper portion of a distribution. The Lognormal fits better over the full distribution.

It is convenient to abbreviate one million and a hundred thousand as M and hk, respectively. We will look at 4 cases of total number of Transactions: 1hk, 2hk, 1M, and 2M.

I. A totally efficient economy. No Taxes or Subsidies. Results in a Pareto Distribution.

30 to 50% in poverty. 4% to 35% have wealth from \$80,000 to \$120,000

II. Subsidies to 30% of the population that have least wealth. The subsidies that are distributed come from taxes alone – Results in runaway Inflation of Money Supply.

7 to 37% in poverty, 6% to 18% have wealth from \$80,000 to \$120,000

III. 1% Tax Rate with a balanced budget (Subsidies are equal to taxes). Increased wealth

About 3% in poverty. 25% to 35% have wealth from \$80,000 to \$120,000

IV. 5% Tax Rate with a balanced budget (Subsidies are equal to taxes).

About ½% in poverty. 53% to 62% have wealth from \$80,000 to \$120,000

V. 20% Tax Rate with a balanced budget (Subsidies are equal to taxes).

Less than 0.3% in poverty. 80% to 90% have wealth from \$80,000 to \$120,000.

Greatest Social Good: Economic Growth Considerations

An important consideration in Social Good is Economic and Technological Growth, the increase in economic output. Our lives today are far better than that of Kings of the distant past. No king of distant past had access to advanced medical technology, health, dentistry, hygiene, transportation, the selection of foods, air conditioning, lighting, communication, TV, computers, modes of entertainment, books, magazines, news sources, education, knowledge of our world and universe, weapons, or military force.

One of the key factors in the above is capital. Capital provides a reservoir of wealth needed to power economic growth.

There have been a number of models based on Thermodynamic Efficiency. Consider this illustration for the need for capital – which provides the “force and energy” for economic and technical growth.

Thermodynamics: Efficiency of Water Wheel

Consider a water wheel. It is a machine for converting the energy of flowing or falling water into useful forms of power. A water wheel consists of a wheel with a number of blades or buckets arranged on the outside rim. The driving force is the weight and pushing pressure of the water flowing into the upper buckets. Water comes in at a certain height and exits at a lower height. If incoming and outgoing streams are at the same height, no work can be done. The efficiency of the wheel is driven by the difference in height between the water coming in at the top and the height of the water flowing out at the bottom. The greater the difference in height, the greater the efficiency of the water wheel. This same principle determines the maximum possible efficiency of an internal combustion engine. The efficiency is determined by the difference of absolute temperatures between the combustion chamber and the temperature of the surrounding environment.

Comment:

Increasing Energy Efficiency has a compounding effect on consumption and CO₂ production. Examples: Innovations such as computer controlled engines and LED lights saved energy. Half of any potential energy savings in buildings can be realized by behavioral changes or small tweaks to equipment scheduling. Energy efficiency may have accounted for 75 percent of U.S. CO₂ reductions in the last few years. Reduced use of cars and warmer winters has reduced heating costs.

This difference in height is an analogy to the wealth or capital needed to drive a business or technological improvement. For example, the money for research into new drugs or the ten billion dollars required for a new microcircuit plant for the next generation of iPhone, computer, TV, self-driving truck, bus, or car, satellites for global communication or GPS, etc. There must be an employer with the capital to motivate a citizen to trade his work for wages. This is true regardless of the desire to use “monetary policy” to effect a more equitable distribution of wealth for its citizens.

What is the trade-off between the “fair” distributions of wealth versus the accumulation of capital needed to drive the water wheel of economic growth efficiently? Monetary Policy must be balanced to meet these competing goals.

Adjusting the economic parameters of our Toy Model reveals both how equitable the resulting distribution of wealth and currency inflation and at the end of case **I.**, it shows the distribution of high end wealth, which can be taken as an analog to capital.

Model Monetary Policy Variables

- Tax Rate
- Progressive Tax Rates – varies with wealth
- How frequently taxes are collected
- Distribution of Subsidies to people based on Wealth
- Percentage of Subsidies distributed across different fractions of society

This Toy Model obviously does not cover all the nuances of an economy, but the blind force of statistics does drive the distribution of wealth. There are some implications:

In what follows, changing the parameters of our statistical engine Toy Model” reveals the following:

1. There must always be a fraction of society that is in a state of “poverty.”
It is unrealistic to expect our world to be a utopia, ours is far from an ideal world.
2. There has to be a Pareto-like distribution to allow for thermodynamic efficiency and to fuel growth.
3. Conundrum: Taxes are needed to provide funds for an equitable distribution (Monetary Policies **III.** to **V.**), but this negates the accumulation of capital needed for growth. As in the case of the water wheel, efficiency increases
4. The money policy has to be loosened to provide the small amount of inflation needed to both provide an equitable distribution of wealth and to fuel investment and growth.
5. Assume 3 transactions per day over a 40-year period.

We will look at the results from the following monetary policy parameter values:

I. No Taxes or Subsidies.

For Purpose of comparison, the two different types of averages were used for this case.

II. Subsidies to 30% of the population that have least wealth

The subsidies that are distributed come from taxes alone.

III. 1% Tax Rate with a balanced budget (Subsidies are equal to taxes)

IV. 5% Tax Rate with a balanced budget (Subsidies are equal to taxes)

V. 20% Tax Rate with a balanced budget (Subsidies are equal to taxes)

Five Cases for "Toy" Models (I. to V.) of Different Monetary Policies: Random Financial Exchanges leads to Exponential (Poverty) and Pareto Wealth Distributions

Simulation of Wealth Accumulation- Random Exchanges: Used Mathcad 14 Math Program



Mathcad Script Available at: VXPhysics.com/EconoPhysics

Rules for Transactions: Three Cases: A: No Taxes/Subsidies, B: Subsidies to Lowest 30%, C. Subsidies to ALL

Given a city of 50,000 people. Everyone is initially given \$100 Thousand. Financial transactions are done between 2 people. They are **N transactions** between random (r n d) pairs of the 50,000 agents. Random integers select a person's ID Number by the function: $\text{trunc}(\text{rnd}(50,000))$. For every transaction, they toss a coin, one person gains a random fraction of money and the other loses that amount. Use Tax/Subsidies to get people out of Poverty. The function **R(N)** makes **N Random** Pairs of bets/exchanges. Transactions between the pairs are not allowed if the bet amount is less than the wealth of either one. When variable **SUBS** is set to 1, The 30% lowest wealth people get Subsidy from Taxes. When **SUBS = 0**, no subsidies.

Tax Ratio (Taxes/Income), TR: Tax%=50% **TR := 0.5** **Set SUBS to 0 to Calculate without Tax/Subsidies:** **SUBS := 1**

Money Dist. N Random Transactions

```

R(N) :=
for n ∈ 0..50000
  Mn ← 100
for m ∈ 0..N
  j ← trunc(rnd(50000))
  k ← trunc(rnd(50000))
  r ← rnd(1)
  Bet ← mean(Mj + Mk) · r
  continue if Mj ≤ Bet ∨ Mk ≤ Bet
  S ← 1 if rnd(1) ≥ 0.5
  S ← -1 otherwise
  Mj ← Mj + S · Bet
  Mk ← Mk - S · Bet
  Tax ← Tax + TR · Bet
  if mod(m, 5000) = 4999 ∧ SUBS = 1
    ML ← submatrix(M, 0, 15000, 0, 0)
    ML ← ML + Tax / 45000
    MH ← submatrix(M, 15001, 50000, 0, 0)
    MH ← MH + Tax / 500000
  M ← stack(ML, MH)
  Tax ← 0
M ← SNN(M)
  
```

Support Functions: Histogram, Sort, # of Back Rows

Back Row M: Sums to S

```

RS(M, S) :=
SM ← csort(M, 0)
Sum ← 0
n ← 50000
while Sum < S - 1
  Sum ← Sum + SMn
  n ← n - 1
50000 - n
  
```

Sort M & No Negative Ms

```

SNN(M) :=
SD ← csort(M, 0)
n ← 0
while SDn < 0
  SDn ← 0
  n ← n + 1
SD
  
```

Histogram of R for \$1000 Divisions

```

(R) :=
n ← 0
for q ∈ 0..120
  hq ← 0
  while trunc(Rn) = q
    hq ← hq + 1
    n ← n + 1
  Xq ← q
hs ← h
  
```

%Population in Poverty, PP15
Pop% Below \$15,000

$$PP_{15}(H) := \frac{\sum_{i=0}^{15} H_i}{500}$$

Total Initial Wealth - \$Thousands, TIW

TIW := 50000 · 100 = 5 × 10⁶

Poverty := 15

Inflation Calculated as Ratio: Present/Initial Wealth

Policy Factors: Competition, Tax Rate, Subsidies -> Wealth Redistribution

Competition - Number of Exchanges: 100,000, 500,000, 2 & 5 Million
Tax Rate (TR) = 50%.

Amount of Subsidy, Division of Taxes Across Income Groups
Size of Population, Frequency of Tax Collection - Redistribution

Results with 50% Tax Redistributed to the Lowest 30% of Population

Poverty: Percent of Pop Less than \$15,000 (15% of Median Wealth)

Percent of Inflation: Inflation Limited to 6, 23, 35, and 65%

Income at of Peak Numbers of Population

Explanation of the Process: Create & Evaluate the Different Distributions

We define k, hk, and M as abbreviations for 1000, 100,000, and 1,000,000

$$k := 1000$$

$$hk := 100000$$

$$M := 1000000$$

$$w := 0..400$$

We start with a city of 50,000 people. Each person initially has 100 \$Thousand. We select a random pair from the 50,000 economic agents (people) and the pair makes a bet. The bet is some random fraction of the average of their wealth. One person gains the amount of the bet and the other loses that amount. The total amount of wealth remains the same 50,000 x 100 \$Thousand. This randomizing pair selection and betting is performed by the above Function R(N). N is the number of random transactions or bets made.

Below we look at the results for four different total transactions. One, 100,000, 1,000,000 and 2,000,000 transactions. The Random Distributions are: $R_1, R_{1hk}, R_{1M}, R_{2M}$. There is also a set of corresponding Histograms, H. After we get the resulting distribution for the the four different numbers of transactions, we make four Histograms $H(R(N))$ of the results of the random transations, R(N). The Histogram calculates the number of people with wealth between 0 and 1 \$thousand, 1 and 2 \$thousand, 2 and 3 \$thousand, etc. For the plot below we display this up to the 120 columns ending with the number of agents who have between 119 and 120 \$thousand.

$$\begin{array}{llll} R_1 := R(1) & R_{1hk} := R(1hk) & R_{1M} := R(1M) & R_{2M} := R(2M) \\ H_1 := H(R_1) & H_{1hk} := H(R_{1hk}) & H_{1M} := H(R_{1M}) & H_{2M} := H(R_{2M}) \end{array}$$

Then we calculate the maximum (max) and modal (mode) values of the distributions R(N) and H(R(N)).

$$\begin{array}{llll} \max(R_1) = 134.83 & \max(R_{1hk}) = 577.58 & \max(R_{1M}) = 1929.19 & \max(R_{2M}) = 2579.67 \\ \max(H_1) = 49999 & \max(H_{1hk}) = 9650 & \max(H_{1M}) = 6144 & \max(H_{2M}) = 13566 \\ \text{mode}(R_1) = 100 & \text{mode}(R_{1hk}) = 100 & \text{mode}(R_{1M}) = 100 & \text{mode}(R_{2M}) = \bullet \end{array}$$

Then we show the number of "Poorest" agents, that is, in the initial, left most, 0 to 1 \$Thousand column.

The modal value for R(N) is the most common value, Initially it is 100 \$Thousand.

$$H_{1100} = 49999 \quad H_{1hk0} = 339 \quad H_{1M0} = 6144 \quad H_{2M0} = 13566$$

Then to get a measure of the "middle class" we compute the number of people with wealth between 80 and 120 \$Thousand and the number of agents with wealth less than 15 \$Thousand.

$P_{80to120}$ is the % of People with Wealth \$80k to \$120k, PP15 is the % of People with Wealth Less than \$15,000

$$\begin{array}{llll} P_{80to120}(H_1) = 100 & P_{80to120}(H_{1hk}) = 33.85 & P_{80to120}(H_{1M}) = 6.39 & P_{80to120}(H_{2M}) = 4.28 \\ PP_{15}(H_1) = 0 & PP_{15}(H_{1hk}) = 9.62 & PP_{15}(H_{1M}) = 44.27 & PP_{15}(H_{2M}) = 58.08 \end{array}$$

Nex we compute a measure of the rate of inflation. We calculate the Total Wealth (T_1) in each of the 4 distributions. We do this by looking at the Ratio between the

Total wealth for that particular distribution to total of the Initial Wealth $TIW = 50,000 \times 100$ \$Thousands.

For the case of No Inflation the Ratio = 1.

$$T_1 := \sum R_1 \quad T_2 := \sum R_{1hk} \quad T_3 := \sum R_{1M} \quad T_4 := \sum R_{2M}$$

Random Exchanges with NO Taxes/Subsidies - Conservation of Money and NO INFLATION

$$\frac{T_1}{TIW} = 1 \quad \frac{T_2}{TIW} = 1 \quad \frac{T_3}{TIW} = 1 \quad \frac{T_4}{TIW} = 1$$

Finally we Plot the Histograms of the 4 Distributions. In I. we also compare this to a Lognormal and Pareto Distribution

Resulting Monetary Statistics in \$Thousands - No Taxes/Subsidies

SUBS := 0

Conservation of Money - NO Taxes/Inflation - Pareto Distribution - Rich get Richer

$k := 1000$	$hk := 100000$	$M := 1000000$	$w := 0..400$	$R_{dot}(N)$
$R_1 := R(1)$	$R_{1hk} := R(1hk)$	$R_{1M} := R(1M)$	$R_{2M} := R(2M)$	
$H_1 := H(R_1)$	$H_{1hk} := H(R_{1hk})$	$H_{1M} := H(R_{1M})$	$H_{2M} := H(R_{2M})$	
$\max(R_1) = 134.83$	$\max(R_{1hk}) = 603.65$	$\max(R_{1M}) = 1701.88$	$\max(R_{2M}) = 3139.23$	
$\max(H_1) = 49999$	$\max(H_{1hk}) = 9646$	$\max(H_{1M}) = 6219$	$\max(H_{2M}) = 13572$	
$\text{mode}(R_1) = 100$	$\text{mode}(R_{1hk}) = 100$	$\text{mode}(R_{1M}) = 100$	$\text{mode}(R_{2M}) = \bullet$	
$H_{1_{100}} = 49999$	$H_{1hk_0} = 362$	$H_{1M_0} = 6219$	$H_{2M_0} = 13572$	

$P_{80to120}$ is the % of People with Wealth \$80k to \$120k, PP_{15} is the % of People with Wealth Less than \$15,000

$P_{80to120}(H_1) = 100$	$P_{80to120}(H_{1hk}) = 33.35$	$P_{80to120}(H_{1M}) = 6.61$	$P_{80to120}(H_{2M}) = 4.26$
$PP_{15}(H_1) = 0$	$PP_{15}(H_{1hk}) = 9.8$	$PP_{15}(H_{1M}) = 44.06$	$PP_{15}(H_{2M}) = 57.98$

$RS(R_{2M}, 0.5TIW) = 2977$ **3002 People have half of the Total Initial Wealth, TIW**

$$T_1 := \sum R_1 \quad T_2 := \sum R_{1hk} \quad T_3 := \sum R_{1M} \quad T_4 := \sum R_{2M}$$

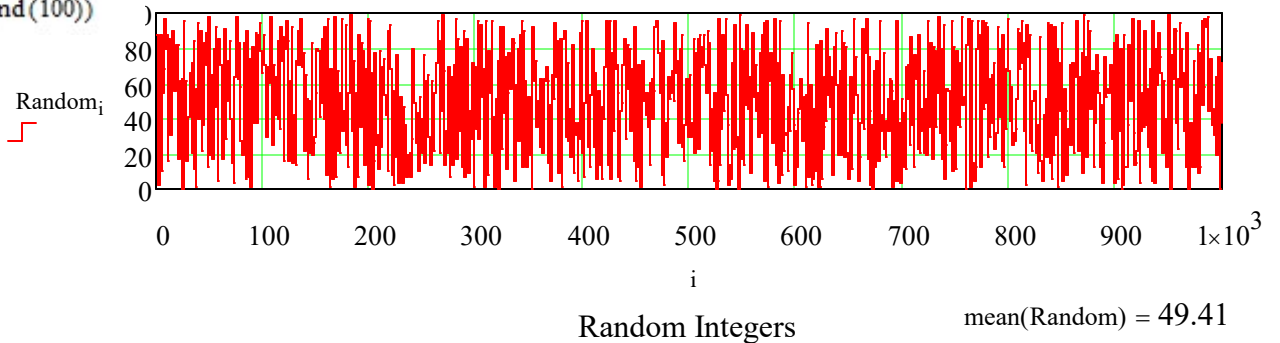
Random Exchanges with NO Taxes/Subsidies - Conservation of Money and NO INFLATION

$\frac{T_1}{TIW} = 1$	$\frac{T_2}{TIW} = 1$	$\frac{T_3}{TIW} = 1$	$\frac{T_4}{TIW} = 1$
$\min(H_1) = 0$	$\min(H_{1hk}) = 141$	$\min(H_{1M}) = 59$	$\min(H_{2M}) = 34$

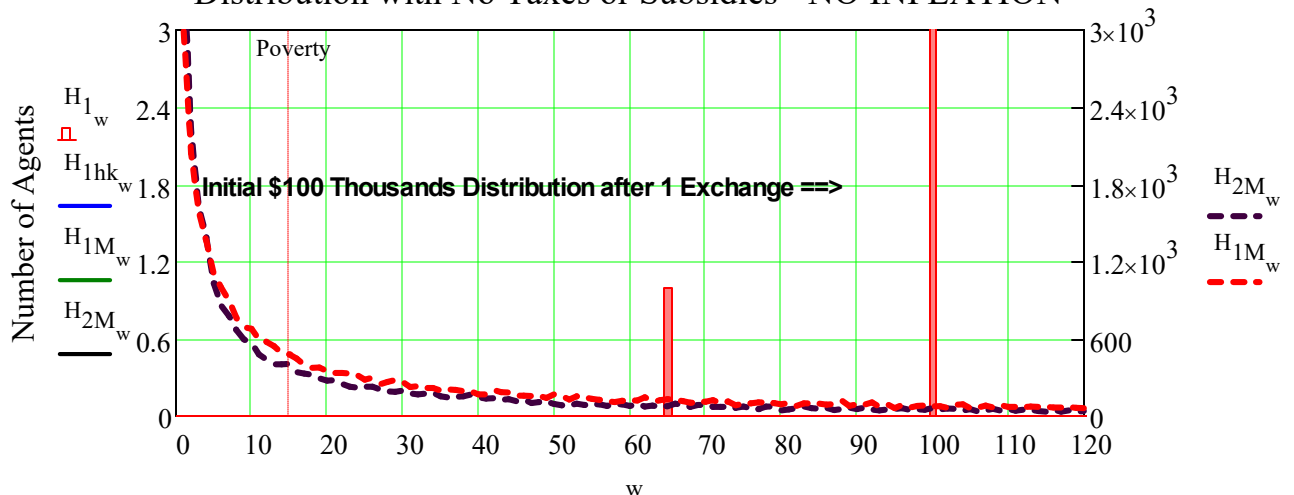
Generate Random Numbers

$i := 0..1000$
 $\text{Random}_i := \text{trunc}(\text{md}(100))$

Random Generation of 1000 Integers that select Agents' ID Numbers



Distribution with No Taxes or Subsidies - NO INFLATION



Redistribution of Wealth - Accumulated Money, $w = \$Thousands$

Match a Log Normal Distribution, LN, to Histogram of Number of Agents vs Wealth, for No Taxes

$$H_{2m} := H_{2M} \quad \text{rows}(H_{2m}) = 121 \quad CC := \text{reverse}(\text{sort}(H_{2m})) \quad ii := 0..120 \quad n_{ii} := ii$$

Log Normal Distribution: $D(n, \alpha, \mu, \sigma) := \text{dlnorm}(n, \mu, \sigma) \cdot \alpha$

Guess for Parameters $\xrightarrow{\text{Given}}$ $\alpha := 10^6 \quad \mu := 3 \quad \sigma := 1.4$
 $DL(\alpha, \mu, \sigma) := CC - D(n, \alpha, \mu, \sigma) \quad 0 = DL(\alpha, \mu, \sigma)$

$$\begin{pmatrix} a \\ m \\ s \end{pmatrix} := \text{Minerr}(\alpha, \mu, \sigma) \quad \begin{pmatrix} a \\ m \\ s \end{pmatrix} = \begin{pmatrix} 39836.93 \\ 2.7 \\ 2.77 \end{pmatrix} \quad \text{LN}(x) := \text{dlnorm}(x, m, s) \cdot a$$

$$\text{LN}(5) = 1059.93$$

Match a Pareto Distribution, Pt, to Histogram of Number of Agents vs Wealth, for No Taxes

Pareto Distribution, Pt: $Pt(x, \alpha, C) := C \cdot \text{if} \left(x \geq 0.0001, \alpha \cdot \frac{0.001^\alpha}{x^{\alpha+1}}, 0 \right) \quad Pto(x, \alpha) := \frac{\alpha}{x^{\alpha+1}}$

H_{2M} is the Histogram for 2 Million Random Transactions, R_{2M} with No Taxes or Subsidies

$$N := \text{rows}(H_{2m}) \quad N = 121 \quad ii := 0..N - 1 \quad x_{ii} := ii \quad \text{Given} \quad \alpha := 1 \quad C := 1$$

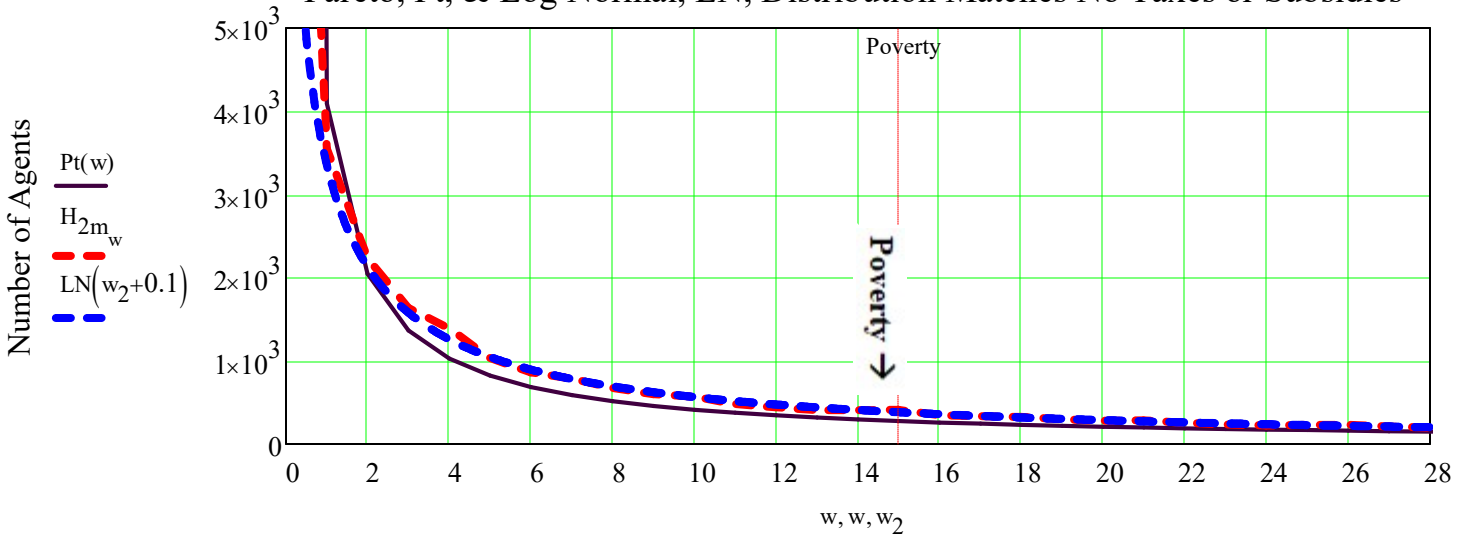
Determine the Coefficients for the Pareto (α, C) that matches Histogram for Random Distribution, H_{2m}

$$\text{resid}(\alpha, C) := H_{2m} - Pt\left(\frac{x}{100}, \alpha, C\right) \quad 0 = \text{resid}(\alpha, C) \quad \text{Poverty} := 15$$

$$\begin{pmatrix} \alpha1 \\ C1 \end{pmatrix} := \text{Minerr}(\alpha, C) \quad \begin{pmatrix} \alpha1 \\ C1 \end{pmatrix} = \begin{pmatrix} 5.91 \times 10^{-8} \\ 6.92 \times 10^8 \end{pmatrix} \quad Pt(x) := Pt\left(\frac{x}{100}, \alpha1, C1\right)$$

$$w_2 := 2w \quad Pt(x) := \text{if}(x < 1, 100000, Pt(x)) \quad Pt(10) = 408.84$$

Pareto, Pt, & Log Normal, LN, Distribution Matches No Taxes or Subsidies



Redistribution of Wealth - Accumulated Money, $w = \$\text{Thousands}$

Match a Log Normal Distribution, LN, to Histogram of Number of Agents vs Wealth, for No Taxes

$$H_{2m} := H_{2M} \quad \text{rows}(H_{2m}) = 121 \quad CC := \text{reverse}(\text{sort}(H_{2m})) \quad ii := 0..120 \quad n_{ii} := ii$$

Log Normal Distribution: $D(n, \alpha, \mu, \sigma) := \text{dlnorm}(n, \mu, \sigma) \cdot \alpha$

Guess for Parameters $\xrightarrow{\text{Given}}$ $\alpha := 10^6 \quad \mu := 3 \quad \sigma := 1.4$

$$DL(\alpha, \mu, \sigma) := CC - D(n, \alpha, \mu, \sigma) \quad 0 = DL(\alpha, \mu, \sigma)$$

$$\begin{pmatrix} a \\ m \\ s \end{pmatrix} := \text{Minerr}(\alpha, \mu, \sigma)$$

$$\begin{pmatrix} a \\ m \\ s \end{pmatrix} = \begin{pmatrix} 33156.55 \\ 1.56 \\ 1.49 \end{pmatrix}$$

$$\text{LN}(x) := \text{dlnorm}(x, m, s) \cdot a$$

$$\text{LN}(5) = 1771.31$$

Match a Pareto Distribution, Pt, to Histogram of Number of Agents vs Wealth, for No Taxes

Pareto Distribution, Pt: $\text{Pt}(x, \alpha, C) := C \cdot \text{if} \left(x \geq 0.0001, \alpha \cdot \frac{0.0001^\alpha}{x^{\alpha+1}}, 0 \right) \quad \text{Pto}(x, \alpha) := \frac{\alpha}{x^{\alpha+1}}$

H_{2M} is the Histogram for 2 Million Random Transactions, R_{2M} with No Taxes or Subsidies

$$\overset{\text{N}}{\text{rows}}(H_{2m}) = N = 121 \quad ii := 0..N-1 \quad x_{ii} := ii \quad \text{Given} \quad \overset{\alpha}{\text{rows}} := 1 \quad \overset{C}{\text{rows}} := 1$$

Determine the Coefficients for the Pareto (α, C) that matches Histogram for Random Distribution, H_{2M}

$$\text{resid}(\alpha, C) := H_{2m} - \text{Pt} \left(\frac{x}{100}, \alpha, C \right) \quad 0 = \text{resid}(\alpha, C)$$

$$\overset{\text{Poverty}}{\text{rows}} := 15$$

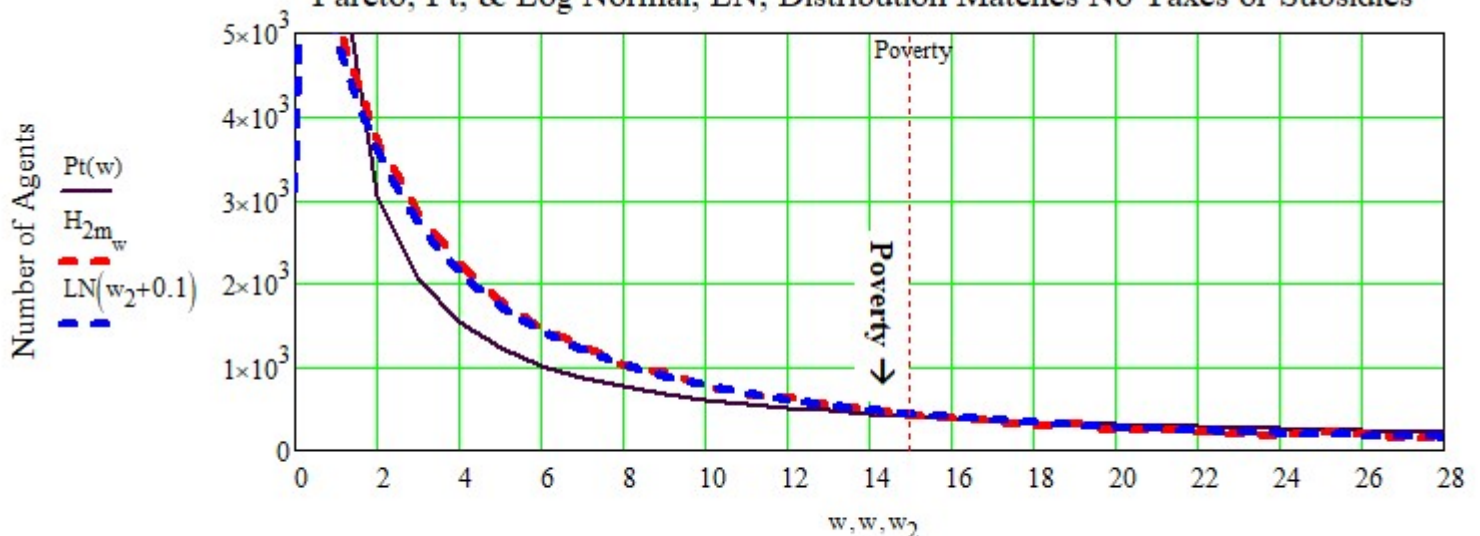
$$\begin{pmatrix} \alpha \\ C \end{pmatrix} := \text{Minerr}(\alpha, C) \quad \begin{pmatrix} \alpha \\ C \end{pmatrix} = \begin{pmatrix} 8.91 \times 10^{-8} \\ 6.88 \times 10^8 \end{pmatrix} \quad \overset{\text{Pt}(x)}{\text{rows}} := \text{Pt} \left(\frac{x}{100}, \alpha, C \right)$$

$$\text{Pt}(10) = 612.96$$

$$w_2 := 2w$$

$$\overset{\text{Pt}(x)}{\text{rows}} := \text{if}(x < 1, 100000, \text{Pt}(x))$$

Pareto, Pt, & Log Normal, LN, Distribution Matches No Taxes or Subsidies



Redistribution of Wealth - Accumulated Money, $w = \$\text{Thousands}$

Histogram of Wealth HW

HW := hist(200, R_{2M})

p := 0..40

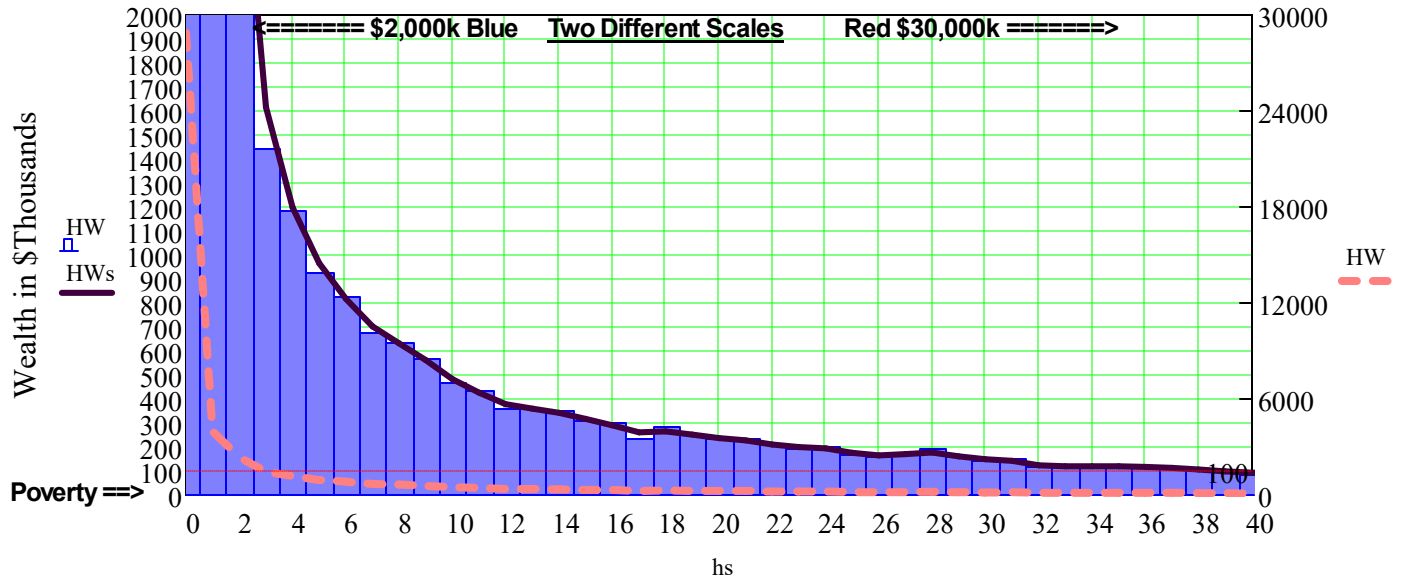
All of the Remaining Plots Use the Average of the Two Agents

Distribution of \$Wealth vs. Number of People- After 2Million Exchanges

40,000 out of 50,000 people have wealth less than their original \$100 Thousand

The top 5 have wealth in Thousands of: \$28,000, \$4,000, \$2,250, \$1,590, \$1,259

Histogram Wealth vs. People Shows Pareto and Log Normal



Redistribution of Wealth - 250 People per Column, 40 = 10,000 People

40 => 10,000 People

II: Resulting Monetary Statistics with Taxes & Subsidies to Lowest 30% Spend More than Collected Taxes Monetary Policy ==> Currency Inflation

SUBS := 1

Function to Find Value of Wealth (n) for Majority of People:

nMax(H) := match(max(H), H)

R_{1hk} := R(1hk)

R_{2hk} := R(2hk)

R_{1M} := R(1M)

R_{2M} := R(2M)

H_{1hk} := H(R_{1hk})

H_{2hk} := H(R_{2hk})

H_{1M} := H(R_{1M})

H_{2M} := H(R_{2M})

max(R_{1hk}) = 663.22

max(R_{2hk}) = 837.56

max(R_{1M}) = 2173.93

max(R_{2M}) = 3710.09

max(H_{1hk}) = 6900

max(H_{2hk}) = 1933

max(H_{1M}) = 1476

max(H_{2M}) = 2076

mode(R_{1hk}) = 101.53

mode(R_{2hk}) = 102.84

mode(R_{1M}) = ■

mode(R_{2M}) = ■

H_{1hk0} = 106

H_{2hk0} = 96

H_{1M0} = 369

H_{2M0} = 674

nMax(H_{1hk}) = (101)

nMax(H_{2hk}) = (102)

nMax(H_{1M}) = (2)

nMax(H_{2M}) = (2)

P_{80to120} is the % of People with Wealth \$80k to \$120k, PP15 is the % of People with Wealth Less than \$15,000

P_{80to120}(H_{1hk}) = 33.95

P_{80to120}(H_{2hk}) = 17.87

P_{80to120}(H_{1M}) = 8.36

P_{80to120}(H_{2M}) = 6.64

PP₁₅(H_{1hk}) = 7.65

PP₁₅(H_{2hk}) = 11.81

PP₁₅(H_{1M}) = 29.09

PP₁₅(H_{2M}) = 36.98

RS(R_{2M}, 0.5TIW) = 1865 **1,876 People have half of the Total Initial Wealth, TIW**

T_{1k} := ∑ R_{1hk}

T_{2k} := ∑ R_{2hk}

T_{3k} := ∑ R_{1M}

T_{4k} := ∑ R_{2M}

T₁ = 5.31 × 10⁶

T₂ = 5.57 × 10⁶

T₃ = 6.91 × 10⁶

T₄ = 8.24 × 10⁶

Random Exchanges with 50% Taxes/Subsidies to Lowest 30%: INFLATION of Currency roughly 6, 23, 35, and 65%.

T₁ / TIW = 1.06

T₂ / TIW = 1.11

T₃ / TIW = 1.38

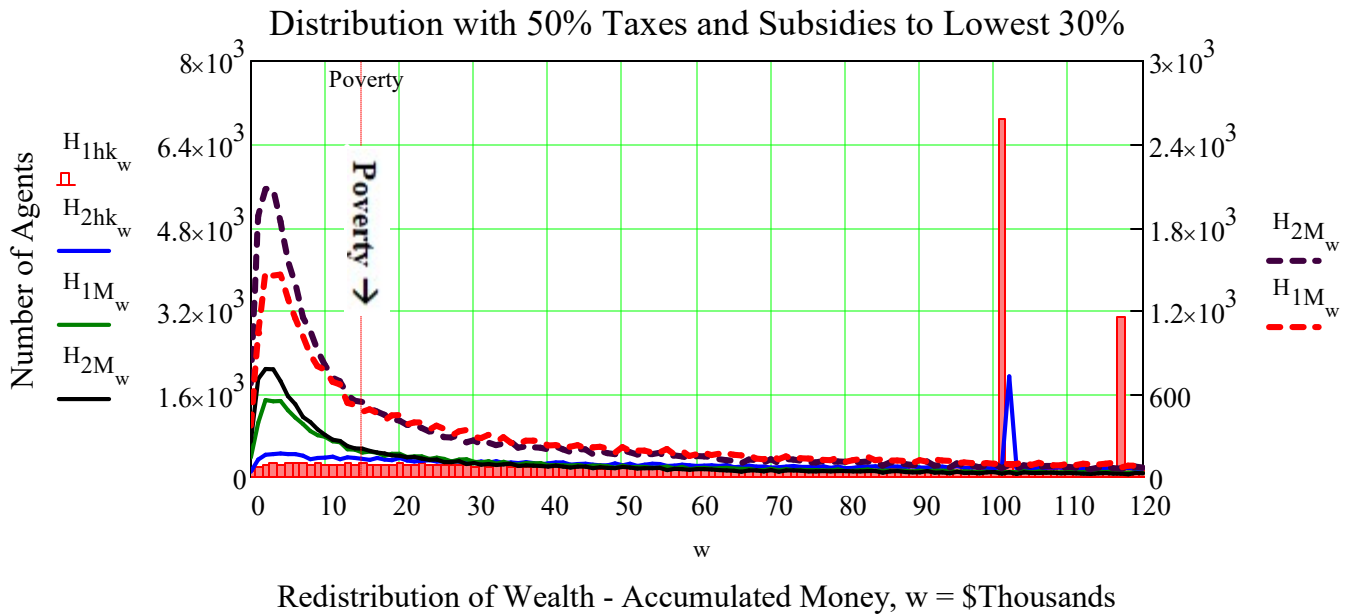
T₄ / TIW = 1.65

min(H_{1hk}) = 106

min(H_{2hk}) = 106

min(H_{1M}) = 68

min(H_{2M}) = 52



Monetary Statistics in \$1000s with 1%/5% Tax and Subsidies to All

Balance the Budget

Equal Tax Shares - Distribution of Collected Taxes to All 50,000 Inhabitants

Resulting Runaway Inflation of 18, 174, 257, and 517 Percent. This would Kill an Economy

Define Exchange Function P(N) with a Tax Rate, TR%, Taxes Equally Divided --> No Inflation

```

P(N, TR) :=
  for n ∈ 0.. 50000
    Mn ← 100
  TF ←  $\frac{TR}{100}$ 
  for m ∈ 0.. N
    j ← trunc(rnd(50000))
    k ← trunc(rnd(50000))
    r ← rnd(1)
    Bet ← mean(Mj + Mk) · r
    continue if Mj ≤ Bet ∨ Mk ≤ Bet
    S ← 1 if rnd(1) ≥ 0.5
    S ← -1 otherwise
    Mj ← Mj + S · Bet
    Mk ← Mk - S · Bet
    M ← M · (1 - TF) +  $\frac{TF \cdot TIW}{50001}$  if mod(m, 1000) = 999
  M ← SNN(M)
  
```

Histogram of P for \$1000 Divisions

```

H(P) :=
  N ← rows(P)
  n ← 0
  Once ← 0
  for q ∈ 0.. 200
    hq ← 0
    while trunc(Pn) = q
      hq ← hq + 1
      n ← n + 1
    Xq ← q
  hs ← ksmooth(X, h, 4)
  
```

III. Monetary Statistics: Tax Rate Only 1% - Balanced Budget- No Inflation

$P_{1hk} := P(1hk, 1)$	$P_{5hk} := P(5hk, 1)$	$P_{1M} := P(1M, 1)$	$P_{2M} := P(2M, 1)$
$H_{1hk} := H(P_{1hk})$	$H_{5hk} := H(P_{5hk})$	$H_{1M} := H(P_{1M})$	$H_{2M} := H(P_{2M})$
$\max(P_{1hk}) = 495.06$	$\max(P_{5hk}) = 636.79$	$\max(P_{1M}) = 600.87$	$\max(P_{2M}) = 856.01$
$\max(H_{1hk}) = 2620.07$	$\max(H_{5hk}) = 393.96$	$\max(H_{1M}) = 406.94$	$\max(H_{2M}) = 416.03$
$\text{mode}(P_{1hk}) = 100$	$\text{mode}(P_{5hk}) = 100$	$\text{mode}(P_{1M}) = \blacksquare$	$\text{mode}(P_{2M}) = \blacksquare$
$\max(H_{1hk}) = 2620.07$	$H_{5hk_0} = 8.82$	$H_{1M_0} = 7.84$	$H_{2M_0} = 20.21$
$n\text{Max}(H_{1hk}) = (99)$	$n\text{Max}(H_{5hk}) = (38)$	$n\text{Max}(H_{1M}) = (64)$	$n\text{Max}(H_{1M}) = (64)$

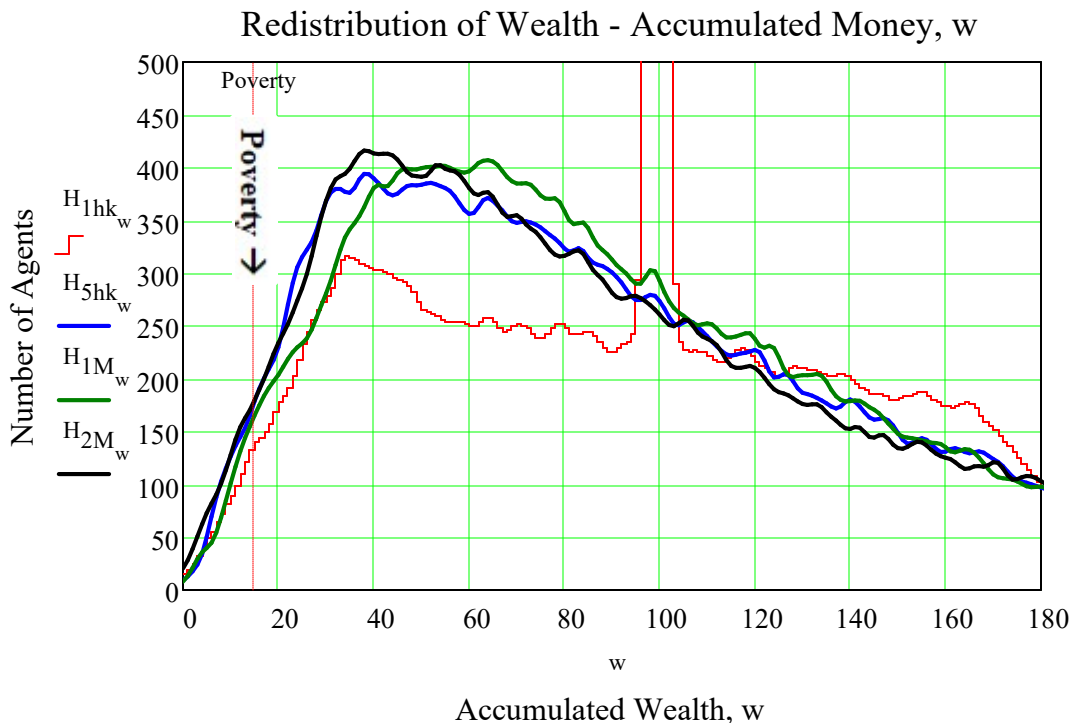
$P_{80to120}$ is the % of People with Wealth \$80k to \$120k, PP_{15} is the % of People with Wealth Less than \$15,000

$P_{80to120}(H_{1hk}) = 36.57$	$P_{80to120}(H_{5hk}) = 22.09$	$P_{80to120}(H_{1M}) = 23.43$	$P_{80to120}(H_{2M}) = 21.54$
$PP_{15}(H_{1hk}) = 2.29$	$PP_{15}(H_{5hk}) = 2.85$	$PP_{15}(H_{1M}) = 2.42$	$PP_{15}(H_{2M}) = 3.19$

$$RS(P_{2M}, 0.5TIW) = 12584 \quad \mathbf{12,939 \text{ People have half of the Total Initial Wealth, TIW}}$$

Random Exchanges with Taxes/Subsidies for All. Run away Inflation 18, 174, 257, and 517 Percent

$TW_1 := \sum P_{1hk}$	$TW_2 := \sum P_{5hk}$	$TW_3 := \sum P_{1M}$	$TW_4 := \sum P_{2M}$
$TW_1 = 5 \times 10^6$	$TW_2 = 5 \times 10^6$	$TW_3 = 5 \times 10^6$	$TW_4 = 5 \times 10^6$
$\frac{TW_1}{TIW} = 1$	$\frac{TW_2}{TIW} = 1$	$\frac{TW_3}{TIW} = 1$	$\frac{TW_4}{TIW} = 1$
$\min(H_{1hk}) = 14.82$	$\min(H_{5hk}) = 8.82$	$\min(H_{1M}) = 7.84$	$\min(H_{2M}) = 20.21$



IV. Monetary Statistics: Tax Rate = 5%

$$P_{1hk} := P(1hk, 5)$$

$$H_{1hk} := H(P_{1hk})$$

$$\max(P_{1hk}) = 327.86$$

$$\max(H_{1hk}) = 2792.79$$

$$\text{mode}(P_{1hk}) = 100$$

$$\max(H_{1hk}) = 2792.79$$

$$n\text{Max}(H_{1hk}) = (99)$$

$$P_{5hk} := P(5hk, 5)$$

$$H_{5hk} := H(P_{5hk})$$

$$\max(P_{5hk}) = 348.13$$

$$\max(H_{5hk}) = 1652.5$$

$$\text{mode}(P_{5hk}) = 100$$

$$H_{5hk_0} = 9.13$$

$$n\text{Max}(H_{5hk}) = (99)$$

$$P_{1M} := P(1M, 5)$$

$$H_{1M} := H(P_{1M})$$

$$\max(P_{1M}) = 340.59$$

$$\max(H_{1M}) = 1663$$

$$\text{mode}(P_{1M}) = \blacksquare$$

$$H_{1M_0} = 4.26$$

$$n\text{Max}(H_{1M}) = (99)$$

$$P_{2M} := P(2M, 5)$$

$$H_{2M} := H(P_{2M})$$

$$\max(P_{2M}) = 386.16$$

$$\max(H_{2M}) = 1157.81$$

$$\text{mode}(P_{2M}) = \blacksquare$$

$$H_{2M_0} = 19.86$$

$$n\text{Max}(H_{2M}) = (99)$$

$P_{80to120}$ is the % of People with Wealth \$80k to \$120k, PP_{15} is the % of People with Wealth Less than \$15,000

$$P_{80to120}(H_{1hk}) = 60.42$$

$$P_{80to120}(H_{5hk}) = 59.71$$

$$P_{80to120}(H_{1M}) = 58.81$$

$$P_{80to120}(H_{2M}) = 47.29$$

$$PP_{15}(H_{1hk}) = 0.19$$

$$PP_{15}(H_{5hk}) = 0.72$$

$$PP_{15}(H_{1M}) = 0.56$$

$$PP_{15}(H_{2M}) = 1.07$$

$$RS(P_{2M}, 0.5TIW) = 17604 \quad \mathbf{20,147 \text{ People have half of the Total Initial Wealth, TIW}}$$

5% TAX RATE RESULTS IN A MORE EQUITABLE DIVISION OF WEALTH

Random Exchanges with Taxes/Subsidies for All. Run away Inflation 18, 174, 257, and 517 Percent

Rate of Inflation is Present Total Wealth (TW) divided by Initial Total Wealth or T/TIW

$$TW_1 := \sum P_{1hk}$$

$$TW_1 = 5 \times 10^6$$

$$\frac{TW_1}{TIW} = 1$$

$$\min(H_{1hk}) = 0.01$$

$$TW_2 := \sum P_{5hk}$$

$$TW_2 = 5 \times 10^6$$

$$\frac{TW_2}{TIW} = 1$$

$$\min(H_{5hk}) = 9.13$$

$$TW_3 := \sum P_{1M}$$

$$TW_3 = 5 \times 10^6$$

$$\frac{TW_3}{TIW} = 1$$

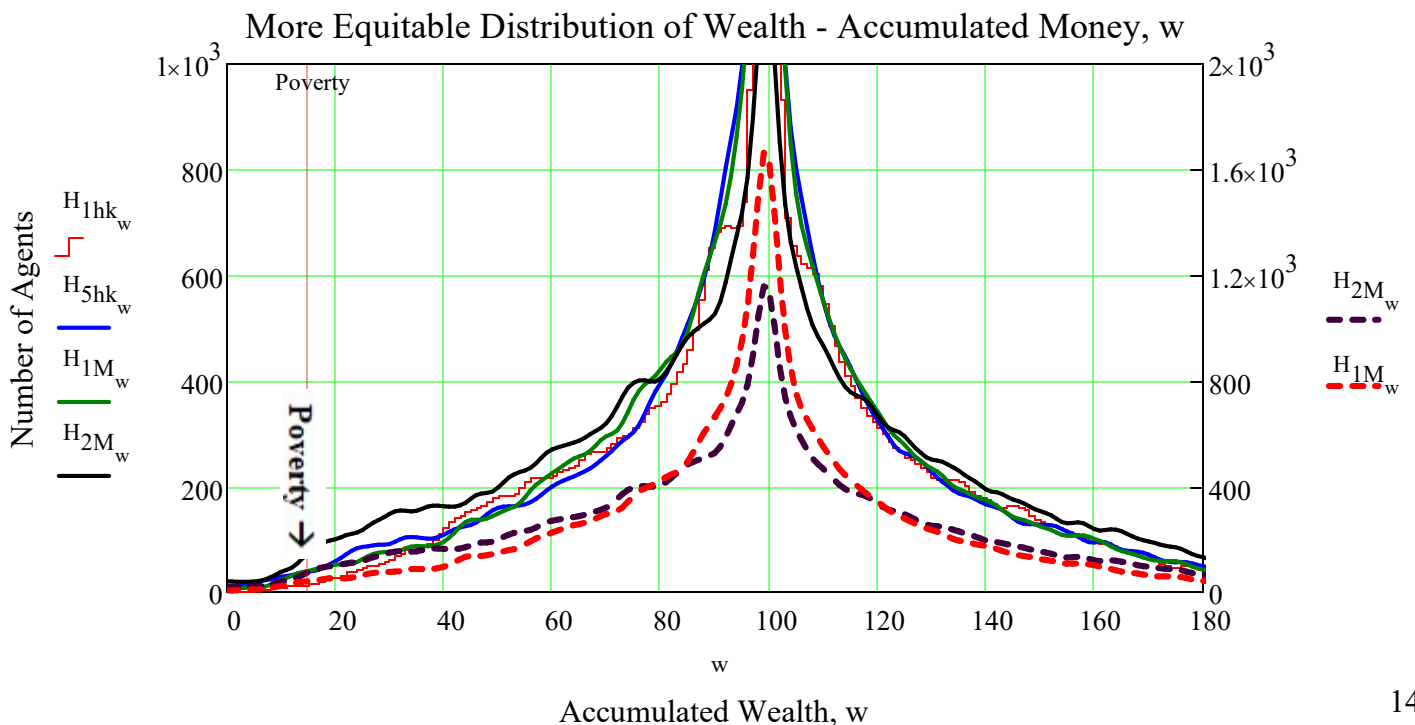
$$\min(H_{1M}) = 4.26$$

$$TW_4 := \sum P_{2M}$$

$$TW_4 = 5 \times 10^6$$

$$\frac{TW_4}{TIW} = 1$$

$$\min(H_{2M}) = 18.77$$



V. Monetary Statistics: Tax Rate = 20%

$P_{1hk} := P(1hk, 20)$	$P_{5hk} := P(5hk, 20)$	$P_{1M} := P(1M, 20)$	$P_{2M} := P(2M, 20)$
$H_{1hk} := H(P_{1hk})$	$H_{5hk} := H(P_{5hk})$	$H_{1M} := H(P_{1M})$	$H_{2M} := H(P_{2M})$
$\max(P_{1hk}) = 275.67$	$\max(P_{5hk}) = 242.81$	$\max(P_{1M}) = 291.25$	$\max(P_{2M}) = 280.22$
$\max(H_{1hk}) = 8729.13$	$\max(H_{5hk}) = 7273.06$	$\max(H_{1M}) = 5954.47$	$\max(H_{2M}) = 6902.01$
$\text{mode}(P_{1hk}) = 100$	$\text{mode}(P_{5hk}) = 100$	$\text{mode}(P_{1M}) = 100$	$\text{mode}(P_{2M}) = 100$
$\max(H_{1hk}) = 8729.13$	$H_{5hk_0} = 5.85$	$H_{1M_0} = 8.56$	$H_{2M_0} = 4$
$n\text{Max}(H_{1hk}) = (99)$	$n\text{Max}(H_{5hk}) = (99)$	$n\text{Max}(H_{1M}) = (99)$	$n\text{Max}(H_{2M}) = (99)$

$P_{80to120}$ is the % of People with Wealth \$80k to \$120k, PP15 is the % of People with Wealth Less than \$15,000

$P_{80to120}(H_{1hk}) = 90.37$	$P_{80to120}(H_{5hk}) = 87.68$	$P_{80to120}(H_{1M}) = 80.85$	$P_{80to120}(H_{2M}) = 87.76$
$PP_{15}(H_{1hk}) = 0.28$	$PP_{15}(H_{5hk}) = 0.21$	$PP_{15}(H_{1M}) = 0.36$	$PP_{15}(H_{2M}) = 0.12$
$RS(P_{2M}, 0.5TIW) = 23083$ 20,147 People have half of the Total Initial Wealth, TIW			

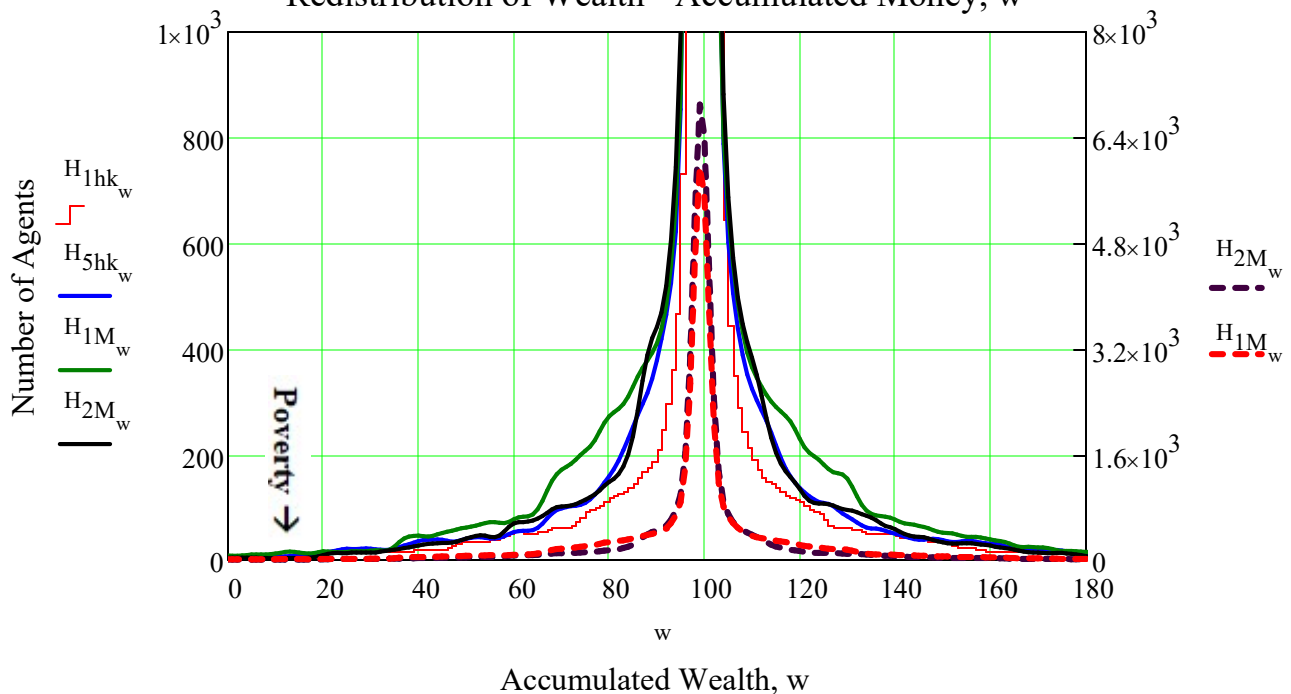
20% TAX RATE RESULTS IN A VERY EVEN (ABOUT 80 TO 90%) DIVISION OF WEALTH

Random Exchanges with Taxes/Subsidies for All. Run away Inflation 18, 174, 257, and 517 Percent

Rate of Inflation is Present Total Wealth (TW) divided by Initial Total Wealth or T/TIW

$TW_1 := \sum P_{1hk}$	$TW_2 := \sum P_{5hk}$	$TW_3 := \sum P_{1M}$	$TW_4 := \sum P_{2M}$
$TW_1 = 5 \times 10^6$	$TW_2 = 5 \times 10^6$	$TW_3 = 5 \times 10^6$	$TW_4 = 5 \times 10^6$
	$\frac{TW_2}{TIW} = 1$	$\frac{TW_3}{TIW} = 1$	$\frac{TW_4}{TIW} = 1$
$\min(H_{1hk}) = 3.89$	$\min(H_{5hk}) = 5.11$	$\min(H_{1M}) = 7.63$	$\min(H_{2M}) = 3.15$
$P_{80to120}(H_{1hk}) = 90.37$	$P_{80to120}(H_{5hk}) = 87.68$	$P_{80to120}(H_{1M}) = 80.85$	$P_{80to120}(H_{2M}) = 87.76$

Redistribution of Wealth - Accumulated Money, w



Conclusions: Monetary Results from Cases I. through V.

I. A totally efficient economy. No Taxes or Subsidies

Pareto and Lognormal Distributions fit to the Random Distribution **I.**

The Distributions for the two different types of average are very similar.

Both of the two different averages can be fitted to Pareto and Lognormal Distributions

30 to 50% in poverty

4% to 35% have wealth from \$80,000 to \$120,000

Distribution of \$Wealth vs. Number of People also has a Lognormal Distribution

II. Subsidies to 30% of the population that have least wealth

The subsidies that are distributed come from taxes alone.

This results in runaway Inflation of Money Supply

Peak of distribution shift to right. Peak now at about \$2,000

7 to 37% in poverty,

6% to 18% have wealth from \$80,000 to \$120,000

III. 1% Tax Rate with a balanced budget (Subsidies are equal to taxes).

Increased wealth

More of an equalitarian distribution

About 3% in poverty

25% to 35% have wealth from \$80,000 to \$120,000

IV. 5% Tax Rate with a balanced budget (Subsidies are equal to taxes)

These parameters result in a bell shaped "equalitarian" distribution.

However, there is less capital for growth and innovation.

About 1/2% in poverty

53% to 62% have wealth from \$80,000 to \$120,000

V. 20% Tax Rate with a balanced budget (Subsidies are equal to taxes)

These parameters result in a bell shaped "equalitarian" distribution.

However, there is less capital for growth and innovation.

Less than 0.3% in poverty

80% to 90% have wealth from \$80,000 to \$120,000.

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