STABILITY CRITERIA FOR FET SWITCHING

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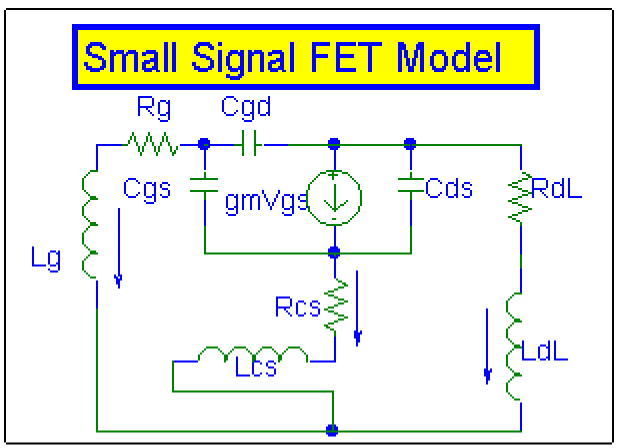
http://www.leapcad.com/Other_Tech/FET_Stability_Analysis.mcd

The behavior of ideal electrical circuit elements and the great majority of engineering applications can be described as Linear Systems. One important aspect of linear system behavior is stability. A system will be unstable if it has any roots in the right half plane. The Routh-Hurwitz criteria for the stability of linear systems states that the necessary condition for asymptotic stability is that both the coefficients of the characteristic polynomial and the Hurwitz determinants be positive.

This criterion can be applied to a MOSFET driving a resistive load, which because of its 100 MHz bandwidth and high input impedance is susceptible to instability. Oscillations can occur when the gate capacitance and circuit board trace inductance form a tens of MHz tank circuit which is not sufficiently damped. What is the criterion for damping? The answer involves 10 variables with very complex relationships. Our goal is to gain insight into these involved relationships. We will develop a tool for this purpose which we will call stability sensitivity.

An analysis of the small signal AC model for a FET is shown below. Since the power supply is ideally a short for AC, the circuit below applies to both high and low side drivers. We shall consider the high side driver case. Rg and Lg are the circuit gate resistance and inductance, respectively.

For the case where a large RFI cap (which behaves as an inductance at high frequencies, i.e. a component of Lg below) shunts the gate to load ground, then Rcs and Lcs are the sum of the source and load resistance and inductance and Rdl and Ldl are simply the FET drain resistance and inductance. For the case where the low end of the RFI cap goes to the source however, then Rcs = 0 and Rdl and Ldl are the sum of the drain plus load resistance and inductance.



The Loop and Node Equations for the above model are:

$Vgs = Vgd + Vds \qquad Ig + Id + Ics = 0$

Vds := 1   Vds := Vds
Vgs := 1   Vgs := Vgs
Vgd := 1   Vgd := Vgd
Ig := 1   Ig := Ig
Ics := 1   Ics := Ics
Id := 1   Id := Id

Cds := 1   Cds := Cds
Cgs := 1   Cgs := Cgs
Lg := 1   Lg := Lg
Rg := 1   Rg := Rg
Cgd := 1   Cgd := Cgd
gm := 1   gm := gm
Lcs := 1   Lcs := Lcs
Ldl := 1   Ldl := Ldl
Rdl := 1   Rdl := Rdl
Rcs := 1   Rcs := Rcs

$d\_dt_{vds} := 1 \qquad d\_dt_{vds} := d\_dt_{vds}$

$d\_dt_{vgs} := 1 \qquad d\_dt_{vgs} := d\_dt_{vgs}$

$d\_dt_{vgd} := 1 \qquad d\_dt_{vgd} := d\_dt_{vgd}$

$d\_dt_{ig} := 1 \qquad d\_dt_{ig} := d\_dt_{ig}$

$d\_dt_{ics} := 1 \qquad d\_dt_{ics} := d\_dt_{ics}$

$d\_dt_{id} := 1 \qquad d\_dt_{id} := d\_dt_{id}$

The above relations were used to substitute for Ics, Vgd and their derivatives, d_dt, in the expressions below.

Denote derivatives by d_dt.

Given

$$Vgs = Ig \cdot Rg + Lg \cdot d\_dt_{ig} + Lcs \cdot \left(d\_dt_{ig} + d\_dt_{id}\right) + Rcs \cdot (Ig + Id)$$

$$Ig = -Cgs \cdot d\_dt_{vgs} - Cgd \cdot \left(d\_dt_{vgs} - d\_dt_{vds}\right)$$

$$Vds = Id \cdot Rdl + Ldl \cdot d\_dt_{id} + Lcs \cdot \left(d\_dt_{ig} + d\_dt_{id}\right) + Rcs \cdot (Ig + Id)$$

$$Cgd \cdot \left(d\_dt_{vgs} - d\_dt_{vds}\right) - gm \cdot Vgs - Cds \cdot d\_dt_{vds} - Id = 0$$

Solve the above for derivative terms, then solve first order simultaneous equations with constant coefficients.

$$\mathrm{Find}\left(d\_dt_{id}, d\_dt_{ig}, d\_dt_{vgs}, d\_dt_{vds}\right) \rightarrow \begin{pmatrix} \dfrac{Lcs\cdot Vds - Lcs\cdot Id\cdot Rdl + Lg\cdot Vds - Lg\cdot Id\cdot Rdl - Lg\cdot Rcs\cdot Ig - Lg\cdot Rcs\cdot Id - Vgs\cdot Lcs + Ig\cdot Rg\cdot Lcs}{Lcs\cdot Ldl + Lg\cdot Ldl + Lg\cdot Lcs} \\ \dfrac{-Ig\cdot Rg\cdot Lcs + Ldl\cdot Vgs + Lcs\cdot Id\cdot Rdl - Lcs\cdot Vds + Vgs\cdot Lcs - Ldl\cdot Ig\cdot Rg - Rcs\cdot Ig\cdot Ldl - Rcs\cdot Id\cdot Ldl}{Lcs\cdot Ldl + Lg\cdot Ldl + Lg\cdot Lcs} \\ \dfrac{-Ig\cdot Cgd - gm\cdot Vgs\cdot Cgd - Cds\cdot Ig - Id\cdot Cgd}{Cgs\cdot Cgd + Cds\cdot Cgs + Cds\cdot Cgd} \\ \dfrac{-Cgs\cdot gm\cdot Vgs - Cgs\cdot Id - Ig\cdot Cgd - gm\cdot Vgs\cdot Cgd - Id\cdot Cgd}{Cgs\cdot Cgd + Cds\cdot Cgs + Cds\cdot Cgd} \end{pmatrix}$$

By inspection, the terms in the above expression can then be grouped as follows to form a matrix equation:

$$\begin{pmatrix} d\_dt_{id} \\ d\_dt_{ig} \\ d\_dt_{vgs} \\ d\_dt_{vds} \end{pmatrix} = A\cdot\begin{pmatrix} Id \\ Ig \\ Vgs \\ Vds \end{pmatrix}$$

Where A is the matrix below.

$$A := \begin{bmatrix} \dfrac{-(Lg + Lcs)\cdot Rdl - Lg\cdot Rcs}{Ldl\cdot Lg + Lcs\cdot Ldl + Lcs\cdot Lg} & \dfrac{Lcs\cdot Rg - Lg\cdot Rcs}{Ldl\cdot Lg + Lcs\cdot Ldl + Lcs\cdot Lg} & \dfrac{-Lcs}{(Ldl\cdot Lg + Lcs\cdot Ldl + Lcs\cdot Lg)} & \dfrac{Lg + Lcs}{Ldl\cdot Lg + Lcs\cdot Ldl + Lcs\cdot Lg} \\ \dfrac{Rdl\cdot Lcs - Ldl\cdot Rcs}{Ldl\cdot Lg + Lcs\cdot Ldl + Lcs\cdot Lg} & \dfrac{-(Ldl + Lcs)\cdot Rg - Ldl\cdot Rcs}{Ldl\cdot Lg + Lcs\cdot Ldl + Lcs\cdot Lg} & \dfrac{Ldl + Lcs}{Ldl\cdot Lg + Lcs\cdot Ldl + Lcs\cdot Lg} & \dfrac{-Lcs}{(Ldl\cdot Lg + Lcs\cdot Ldl + Lcs\cdot Lg)} \\ \dfrac{-Cgd}{Cds\cdot Cgs + Cds\cdot Cgd + Cgd\cdot Cgs} & \dfrac{-(Cgd + Cds)}{Cds\cdot Cgs + Cds\cdot Cgd + Cgd\cdot Cgs} & \dfrac{-Cgd\cdot gm}{Cds\cdot Cgs + Cds\cdot Cgd + Cgd\cdot Cgs} & 0 \\ \dfrac{-(Cgs + Cgd)}{(Cds\cdot Cgs + Cds\cdot Cgd + Cgd\cdot Cgs)} & \dfrac{-Cgd}{(Cds\cdot Cgs + Cds\cdot Cgd + Cgd\cdot Cgs)} & \dfrac{-(Cgs + Cgd)\cdot gm}{Cds\cdot Cgs + Cds\cdot Cgd + Cgd\cdot Cgs} & 0 \end{bmatrix}$$

The expressions of A can be compacted by defining the terms in the denominators as Leff and Ceff

$$Leff := \sqrt{Ldl\cdot Lg + Lcs\cdot Ldl + Lcs\cdot Lg} \qquad Ceff := \sqrt{Cds\cdot Cgs + Cds\cdot Cgd + Cgd\cdot Cgs}$$

$$A := \begin{bmatrix} \frac{-(Lg + Lcs)\cdot Rdl - Lg\cdot Rcs}{Leff^{2}} & \frac{Lcs\cdot Rg - Lg\cdot Rcs}{Leff^{2}} & \frac{-Lcs}{Leff^{2}} & \frac{Lg + Lcs}{Leff^{2}} \\ \frac{Rdl\cdot Lcs - Ldl\cdot Rcs}{Leff^{2}} & \frac{-(Ldl + Lcs)\cdot Rg - Ldl\cdot Rcs}{Leff^{2}} & \frac{Ldl + Lcs}{Leff^{2}} & \frac{-Lcs}{Leff^{2}} \\ \frac{-Cgd}{Ceff^{2}} & \frac{-(Cgd + Cds)}{Ceff^{2}} & \frac{-Cgd\cdot gm}{Ceff^{2}} & 0 \\ \frac{-(Cgs + Cgd)}{Ceff^{2}} & \frac{-Cgd}{Ceff^{2}} & \frac{-(Cgs + Cgd)\cdot gm}{Ceff^{2}} & 0 \end{bmatrix}$$

The above set of simultaneous differential equations is linear with constant coefficients. These equations have solutions of the form C exp(λt) and they can be solved algebraically by finding the solution to the characteristic equation, which is the determinant of "A - λI", where I is the 4 x 4 Identity matrix.

$I := \text{identity}(4)$

Evaluate the determinant and collect the terms in λ below.

$$|A - \lambda \cdot I| \text{ collect}, \lambda \rightarrow \frac{Cds \cdot Cgd \cdot Lg \cdot Lcs + Cds \cdot Cgd \cdot Lg \cdot Ldl + Cgs \cdot Cgd \cdot Lg \cdot Lcs + Cgs \cdot Cgd \cdot Lg \cdot Ldl + Cgs \cdot Cgd \cdot Lcs \cdot Ldl + Cds \cdot Cgd \cdot Lcs \cdot Ldl + Cds \cdot Cgs \cdot Lg \cdot Ldl + Cds \cdot Cgs \cdot Lcs \cdot Ldl + Cds \cdot C}{(Lcs \cdot Ldl + Lg \cdot Ldl + Lg \cdot Lcs) \cdot (Cgs \cdot Cgd + Cds \cdot Cgs + Cds \cdot Cgd)}$$

The above equation is 10 pages wide. It is of the form: $a_4 \cdot \lambda^4 + a_3 \lambda^3 + a_2 \cdot \lambda^2 + a_1 \cdot \lambda + a_o = 0$

This 4th degree equation does not have a closed form solution. We will deal with it by numeric means.
The equation can be compacted by gathering like terms and extracting common terns, Leff and Ceff.
After dropping the common denominator, we copy over the last coefficients a1 and ao. They are:

$$a_1 = (Rcs + Rdl) \cdot Cds + Lcs \cdot gm + (Rdl + Rg) \cdot Cgd + (Rg + Rcs) \cdot Cgs + (Rdl \cdot Rg + Rcs \cdot Rg) \cdot gm\, Cgd + Rdl \cdot Rcs \cdot Cgd \cdot gm$$

Gain Factor: $a_o = 1 + Rcs \cdot gm$

The first 9 terms of the above expression for $|A-\lambda I|$ form the coefficient a4 of λ^4. By inspection we see that they are product terms of $Leff^2$ and $Ceff^2$, i..e.

$$(Ldl \cdot Lg + Lcs \cdot Ldl + Lcs \cdot Lg) \cdot (Cds \cdot Cgs + Cds \cdot Cgd + Cgd \cdot Cgs)$$

Then a4 = $Leff^2$ x $Ceff^2$.

Coefficients a3 and a2 are found similarly and are given below.

WITH GATE RFI CAP TO GND, WHAT ARE THE VALUES OF THE FET AND CIRCUIT PARAMETERS?

Cgs is voltage independent. The voltage dependence of Cgs and Cds flattens out when Vgs > 3V and Vds > 6V , respectively.
Hot bulb 10 ohm, I limit ~ 4A, gm ~ 2 S

MOTO MTP3055VL DATA SHEET: **ESTIMATES OF COD CIRCUIT PARAMETERS**

$Ciss := 410 \cdot pF$ $\quad Coss := 114 \cdot pF$ $\quad L(mm) := 1 \cdot nH \cdot mm$ $\quad Cg := 1 \cdot Ciss$

$Crss := 21 \cdot pF$ $\quad Qt := 8.1 \cdot nC$ $\quad Cgs := 0.8 \cdot Ciss + 100 \cdot pF$ $\quad Cgd := 0.2 \cdot Ciss + 10 \cdot pF$

$Ld := 3.5 \cdot nH$ $\quad Ls := 7.5 \cdot nH$ $\quad Cds := 1 \cdot Coss + 100 \cdot pF$ $\quad Rg := 6.5 \cdot \Omega$

$g_{FS} := 8.8 \cdot S$ $\quad Rdson := 0.12 \cdot \Omega$ $\quad gm := 2 \cdot S$ $\quad Lg := 30 \cdot nH$

$tr := 85 \cdot nsec$ $\quad tf := 43 \cdot nsec$ $\quad Ldl := Ld + 50 \cdot nH$ $\quad Lcs := Ls + 10 \cdot nH$

$Vt := 1.6 \cdot volt$ $\quad Rds := 1 \cdot \Omega$ $\quad Rload := 10 \cdot \Omega$ $\quad Rdl := Rds$

$Rcs := Rload$

$Leff := \sqrt{Ldl \cdot Lg + Lcs \cdot Ldl + Lcs \cdot Lg}$ $\quad Ceff := \sqrt{Cds \cdot Cgs + Cds \cdot Cgd + Cgd \cdot Cgs}$

$a_4 := Ceff^2 \cdot Leff^2 \cdot sec^{-4}$

$a_3 := (Rdl \cdot Lg \cdot Cds \cdot Cgd + Ldl \cdot Lcs \cdot gm \cdot Cgd + Ldl \cdot gm \cdot Lg \cdot Cgd + Lcs \cdot Rdl \cdot Cds \cdot Cgd + Cgs \cdot Cds \cdot Ldl \cdot Rcs + Cgs \cdot Cds \cdot Lcs \cdot Rdl + Cgs \cdot Ldl \cdot Rcs \cdot Cgd + Cgs \cdot Cds \cdot Rg \cdot Lcs + Ldl \cdot Cds \cdot Rcs \cdot Cgd + C$

$a_2 := (Ldl \cdot Cds + Lcs \cdot Cgs + Lg \cdot Cgs + Cgs \cdot Rdl \cdot Rcs \cdot Cgd + Cgs \cdot Rcs \cdot Rg \cdot Cgd + Lcs \cdot Cds + Cgs \cdot Cds \cdot Rcs \cdot Rg + Cgs \cdot Cds \cdot Rdl \cdot Rg + Cgs \cdot Rdl \cdot Rg \cdot Cgd + Rdl \cdot Cds \cdot Rg \cdot Cgd + Rdl \cdot Lg \cdot gm \cdot Cgd +$

$a_1 := (Rcs \cdot Cds + Lcs \cdot gm + Rdl \cdot Cgd + Rg \cdot Cgs + Rdl \cdot Rg \cdot gm \cdot Cgd + Rg \cdot Cgd + Cds \cdot Rdl + Rcs \cdot Cgs + Rdl \cdot Rcs \cdot Cgd \cdot gm + Rcs \cdot Rg \cdot gm \cdot Cgd) \cdot sec^{-1}$

$a_o := 1 + Rcs \cdot gm$ $\quad a_4 = 4.619 \times 10^{-3}$ $\quad a_3 = 7.667 \times 10^{-25}$ $\quad a_2 = 3.028 \times 10^{-16}$ $\quad a_1 = 6.01 \times 10^{-8}$ $\quad a_o = 21$

Solution for the Given Parameters

Find the roots, λ, numerically, for the coefficients a_x of characteristic equation, f(λ) = 0, given the particular set of parameters given above:

$v := \left(a_o \ \ a_1 \ \ a_2 \ \ a_3 \ \ a_4\right)^T$

$f(\lambda) := a_4 \cdot \lambda^4 + a_3 \cdot \lambda^3 + a_2 \cdot \lambda^2 + a_1 \cdot \lambda + a_o$

solution := polyroots(v)

$$\text{solution} = \begin{pmatrix} -1.164 \times 10^9 \\ -4.968 \times 10^8 \\ 3.971 \times 10^5 - 2.804i \times 10^8 \\ 3.971 \times 10^5 + 2.804i \times 10^8 \end{pmatrix}$$

The natural frequency jω = 2 π freq

$\text{freq} := \dfrac{\text{Im}\left(\text{solution}_3\right)}{2 \cdot \pi}$ $\text{freq} = -4.463 \times 1\upsilon$

$f_{eff} := \dfrac{1}{2 \cdot \pi} \cdot \dfrac{1}{\sqrt{\text{Leff} \cdot \text{Ceff}}}$

$f_{eff} = 3.433 \times 10^7 \ \text{Hz}$

CHARACTERISTICS OF SOLUTIONS:

Because the coefficients of the characteristic equation, a1 through a4, are so small (on the order of {nF x nH}^{2}), the roots, which we denote as σ + jω, must be very large (in the negative direction) so that they sum up to - ao, where ao ~ 317. Root locus is not very helpful as an analysis technique because of the huge difference in the size of the roots (10^9) versus the gain term, ao.

The **solution is unstable** because the real parts of the last two roots are positive. For analysis of stability, we are most interested in the behavior of the last two complex roots, which have the positive real part. The jω portion is larger than the real part. These roots have both upper+ and lower- branches. **The natural frequency of the instability is equal to 44 MHz.**

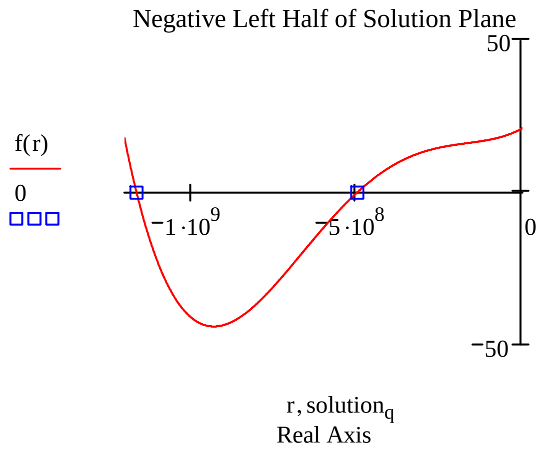
PLOT ON REAL AXIS, R

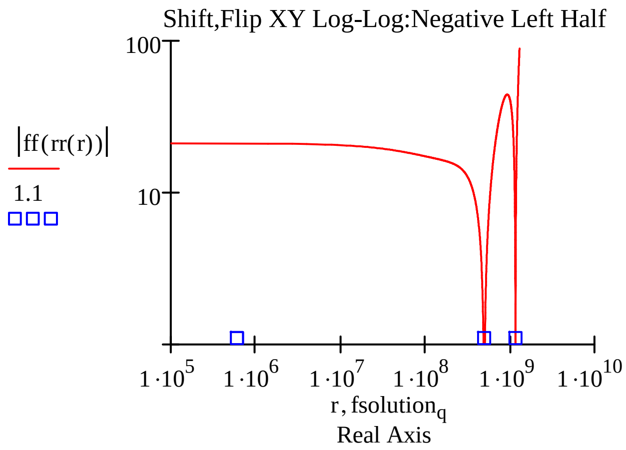
From the plot of the curve on the left below, we see that three of the solutions are bunched together into what appears to be a single square at the origin. To spread out this tight range of values, we wish to display the curve as a log-log plot. This requires positive x and y values. We need to flip both the x and y axes. Redefine λ -> -λ, ff = |f(λ)|, multiply the "solution" by -1 and then plot the solution zeros as boxes. To see the real part, r, (j ω = 0) of the positive unstable solution, shift the plot right by 1 10^6. This gives the original and shifted log-log plots below.

$\text{fsolution} := \text{Re}\left(\left(-1 \cdot \text{solution} + 10^6\right)\right)$

$ff(\lambda) := a_4 \cdot (-\lambda)^4 + a_3 \cdot (-\lambda)^3 + a_2 \cdot (-\lambda)^2 + a_1 \cdot -\lambda + a_o$ $q := 1 .. 4$ $rr(r) := r - 10^6$

GRAPHICAL DISPLAY AND VERIFICATION OF THE REAL PART OF ROOTS





$f(\lambda_3) \sim 0$: $f(solution_3) = 1.386 \times 10^{-9} + 1.537i \times 10^{-10}$

Because of the complexity of the dependency and the inter-relation of the roots on the circuit parameters, we gain very little insight about how the stability is affected by the variation of circuit parameters. **We will try the classic textbook stability analysis.**

Classical Analysis:The Routh Stability Criterion

***Routh's Stability Criterion*: the number of real positive roots is equal to the number of changes in sign in column 1 of the Routh Array.**
Calculate the Routh parameters. See for example, D'azzo and Houpis, "Feedback Control System Analysis and Synthesis", pg.121.

$c1 := \frac{a_3 \cdot a_2 - a_4 \cdot a_1}{a_3}$ $\quad c2 := \frac{a_3 \cdot a_o}{a_3}$ $\quad d1 := \frac{c1 \cdot a_1 - a_3 \cdot c2}{c1}$ $\quad e1 := \frac{c2 \cdot a_o}{c2}$

$$RouthArray := \begin{pmatrix} a_4 & a_2 & a_o \\ a_3 & a_1 & 0 \\ c1 & c2 & 0 \\ d1 & 0 & 0 \\ e1 & 0 & 0 \end{pmatrix} \qquad RouthArray = \begin{pmatrix} 4.619 \times 10^{-34} & 3.028 \times 10^{-16} & 21 \\ 7.667 \times 10^{-25} & 6.01 \times 10^{-8} & 0 \\ 2.666 \times 10^{-16} & 21 & 0 \\ -2.96 \times 10^{-10} & 0 & 0 \\ 21 & 0 & 0 \end{pmatrix}$$

There is a change of sign: Therefore there are roots in the right hand plane.
The response function is unstable, i.e. not bounded in time.

Hurwitz Stability Criterion

Polynomials whose zeros have negative real parts are Hurwitz polynomials. The Hurwitz test for stability is that the Hurwitz determinants, Dx, be greater than zero.

The Hurwitz Determinants, D2, D3, D4, for a fourth order polynomial are:

$$D2 := \left| \begin{pmatrix} a_3 & a_1 \\ a_4 & a_2 \end{pmatrix} \right| \qquad D3 := \left| \begin{pmatrix} a_3 & a_1 & 0 \\ a_4 & a_2 & a_o \\ 0 & a_3 & a_1 \end{pmatrix} \right| \qquad D4 := \left| \begin{pmatrix} a_3 & a_1 & 0 & 0 \\ a_4 & a_2 & a_o & 0 \\ 0 & a_3 & a_1 & 0 \\ 0 & a_4 & a_2 & a_o \end{pmatrix} \right|$$

Calculate the Hurwitz Determinants. For stability they must be positive.

$$D2 = 2.044 \times 10^{-40} \qquad D3 = -6.05 \times 10^{-50} \qquad D4 = -1.27 \times 10^{-48}$$

For the parametric conditions given above, **D3 is negative and the circuit is unstable**.

We are no better off than before. We still need some strategy to relate the circuit variables to the onset of instability. By trying different parameter values we find that stability is lost only when d1 and D3 go negative. Expanding d1 & D3 reveals that they are equivalent and equal to $a_3 \cdot a_2 \cdot a_1 - a_3^2 \cdot a_o - a_4 \cdot a_1^2$. This tells us that **instability occurs when**

$$a_3^2 \cdot a_o + a_4 \cdot a_1^2 > a_3 \cdot a_2 \cdot a_1$$

METHODOLOGY: This gives us a criterion for instability, but we still need a methodology to gain insight into how the relationships among the circuit parameters affect stability. For this purpose, we will develop and use the concept of **stability sensitivity**, which is the rate of change of D3 with respect to the circuit parameters. We can then rank the parameters and observe how the direction and magnitude of this sensitivity change with the highest ranking factor.

STRATEGY: DETERMINE THE DIRECTION AND MAGNITUDE OF SENSITIVITY TO INSTABILITY AS FUNCTION OF PARAMETERS

Define Dimensionless Parameters (F, H, Ohm, Siemens)

$p := 10^{-12}$ $n := 10^{-9}$

DIMENSIONLESS VALUES OF THE FET AND CIRCUIT PARAMETERS

Hot bulb 10 ohm, I limit ~ 4A, gm ~ 2

MOTO MTP3055VL DATA SHEET:

$Ciss := 410\cdot p$ $Coss := 114\cdot p$
$Crss := 21\cdot p$ $Qt := 8.1\cdot nC$
$Ld := 3.5\cdot n$ $Ls := 7.5\cdot n$
$g_{FS} := 8.8$ $Rdson := 0.12$
$tr := 85\cdot nsec$ $tf := 43\cdot nsec$
$Vt := 1.6\cdot volt$ $Rds := 1$

ESTIMATES OF COD CIRCUIT PARAMETERS

$L(mm) := 1\cdot n\cdot mm$ $Cg := 1\cdot Ciss$
$Cgs := 0.8\cdot Ciss + 100\cdot p$ $Cgd := 0.2\cdot Ciss + 10\cdot p$
$Cds := 1\cdot Coss + 100\cdot p$ $Rg := 6.5$
$gm := 2$ $Lg := 30\cdot n$
$Ldl := 50\cdot n + Ld$ $Lcs := Ls + 10\cdot n$
$Rload := 10$ $Rdl := Rds$
$Rcs := Rload$

Evaluate Stability as function of the 10 variables Cgs, Cgd, Cds, Rg, Lg, Rdl, Ldl, Rcs, Lcs and gm.

$a_4(Cgs, Cgd, Cds, Lg, Ldl, Lcs) := (Ldl\cdot Lg + Lcs\cdot Ldl + Lcs\cdot Lg)\cdot(Cds\cdot Cgs + Cds\cdot Cgd + Cgd\cdot Cgs)$

$a_3(Cgs, Cgd, Cds, Rg, Lg, Rdl, Ldl, Rcs, Lcs, gm) := (Rdl\cdot Lg\cdot Cds\cdot Cgd + Ldl\cdot Lcs\cdot gm\cdot Cgd + Ldl\cdot gm\cdot Lg\cdot Cgd + Lcs\cdot Rdl\cdot Cds\cdot Cgd + Cgs\cdot Cds\cdot Ldl\cdot Rcs + Cgs\cdot Cds\cdot Lcs\cdot Rdl + Cgs\cdot Ldl\cdot Rcs\cdot C$

$a_2(Cgs, Cgd, Cds, Rg, Lg, Rdl, Ldl, Rcs, Lcs, gm) := (Ldl\cdot Cds + Lcs\cdot Cgs + Lg\cdot Cgs + Cgs\cdot Rdl\cdot Rcs\cdot Cgd + Cgs\cdot Rcs\cdot Rg\cdot Cgd + Lcs\cdot Cds + Cgs\cdot Cds\cdot Rcs\cdot Rg + Cgs\cdot Cds\cdot Rdl\cdot Rg + Cgs\cdot Rdl\cdot$

$a_1(Cgs, Cgd, Cds, Rg, Lg, Rdl, Rcs, Lcs, gm) := (Rcs\cdot Cds + Lcs\cdot gm + Rdl\cdot Cgd + Rg\cdot Cgs + Rdl\cdot Rg\cdot gm\cdot Cgd + Rg\cdot Cgd + Cds\cdot Rdl + Rcs\cdot Cgs + Rdl\cdot Rcs\cdot Cgd\cdot gm + Rcs\cdot Rg\cdot gm\cdot Cgd)$

$a_o(Rcs, gm) := 1 + Rcs\cdot gm$ $a_4(Cgs, Cgd, Cds, Lg, Ldl, Lcs) = 4.619\times 10^{-34}$

Define Hurwitz Criteria, d3, as a function of Cgs, Cgd, Cds, Rg, Lg, Rdl, Ldl, Rcs, Lcs and gm.

$$D3 = a_3\cdot a_2\cdot a_1 - a_3^{\,2}\cdot a_o - a_4\cdot a_1^{\,2}$$

$d3(Cgs, Cgd, Cds, Rg, Lg, Rdl, Ldl, Rcs, Lcs, gm) := a_3(Cgs, Cgd, Cds, Rg, Lg, Rdl, Ldl, Rcs, Lcs, gm)\cdot a_2(Cgs, Cgd, Cds, Rg, Lg, Rdl, Ldl, Rcs, Lcs, gm)\cdot a_1(Cgs, Cgd, Cds, Rg, Lg, Rdl, Rcs$
$+ -a_3(Cgs, Cgd, Cds, Rg, Lg, Rdl, Ldl, Rcs, Lcs, gm)^2\cdot a_o(Rcs, gm) - a_4(Cgs, Cgd, Cds, Lg, Ldl, Lcs)\, a_1(Cgs, Cgd, Cds, Rg, Lg, Rdl, Rc$

Check Results: d3 vs D3: $d3(Cgs, Cgd, Cds, Rg, Lg, Rdl, Ldl, Rcs, Lcs, gm) = -6.05\times 10^{-50}$ $D3 = -6.05\times 10^{-50}$

Roots of Characteristic Equation

$vv := (a_o(Rcs, gm)\ \ a_1(Cgs, Cgd, Cds, Rg, Lg, Ldl, Rcs, Lcs, gm)\ \ a_2(Cgs, Cgd, Cds, Rg, Lg, Rdl, Ldl, Rcs, Lcs, gm)\ \ a_3(Cgs, Cgd, Cds, Rg, Lg, Rdl, Ldl, Rcs, Lcs, gm)\ \ a_4(Cgs, Cgd, Cds,$

$roots := polyroots(vv)$ $\quad freq := \frac{Im(roots_3)}{2 \cdot \pi}$ $\quad freq = 4.406 \times 10^7$

STABILITY SENSITIVITY ANALYSIS @Estimated Ckt Parameters

Sensitivity Ranking @ECP: Cgd, Cds, Cgs, Lg, Lcs, Ldl, gm, Rdl, Rg, Rcs.
From the plot below, the only parameters that always damp* are Rg & Rdl.
Sensitivity sign*, magnitude & ranking changes with the parameter values.
In particular, the sign of Lg varies with the magnitude of other parameters.
Rg & Rdl are < other sensitivities, but dominate because others flip signs.
The Rg effect is very similar to that of Rdl. The greatest change is for Cgd.
The effect of Cgd is a factor of 10^{10} or more larger than Rg, Rdl, gm & Rcs.
The size or the effect of a sensitivity decreases with its relative magnitude.
Lcs is critical in affecting the natural frequency, band width.

|RANKING|, MAGNITUDES AND SIGNS OF STABILITY SENSITIVITIES

$$\frac{d}{dCgd} d3(Cgs, Cgd, Cds, Rg, Lg, Rdl, Ldl, Rcs, Lcs, gm) = 5.517 \times 10^{-38}$$

$$\frac{d}{dCds} d3(Cgs, Cgd, Cds, Rg, Lg, Rdl, Ldl, Rcs, Lcs, gm) = -8.379 \times 10^{-39}$$

$$\frac{d}{dCgs} d3(Cgs, Cgd, Cds, Rg, Lg, Rdl, Ldl, Rcs, Lcs, gm) = -2.896 \times 10^{-39}$$

$$\frac{d}{dLg} d3(Cgs, Cgd, Cds, Rg, Lg, Rdl, Ldl, Rcs, Lcs, gm) = -1.275 \times 10^{-40}$$

$$\frac{d}{dLcs} d3(Cgs, Cgd, Cds, Rg, Lg, Rdl, Ldl, Rcs, Lcs, gm) = 1.7 \times 10^{-40}$$

$$\frac{d}{dLdl} d3(Cgs, Cgd, Cds, Rg, Lg, Rdl, Ldl, Rcs, Lcs, gm) = -2.905 \times 10^{-41}$$

$$\frac{d}{dgm} d3(Cgs, Cgd, Cds, Rg, Lg, Rdl, Ldl, Rcs, Lcs, gm) = 2.844 \times 10^{-49}$$

$$\frac{d}{dRg} d3(Cgs, Cgd, Cds, Rg, Lg, Rdl, Ldl, Rcs, Lcs, gm) = 9.778 \times 10^{-49}$$

$$\frac{d}{dRdl} d3(Cgs, Cgd, Cds, Rg, Lg, Rdl, Ldl, Rcs, Lcs, gm) = 1.007 \times 10^{-48}$$

$$\frac{d}{dRcs}d3(Cgs, Cgd, Cds, Rg, Lg, Rdl, Ldl, Rcs, Lcs, gm) = -2.346\times 10^{-49}$$

Define a Stability Function, DRL, as a function of some +/- Sensitivity pairs

$$DRL(Rg, Rdl, Lg, Rcs, Lcs, gm) := (d3(Cgs, Cgd, Cds, Rg, Lg, Rdl, Ldl, Rcs, Lcs, gm))\cdot 10^{47}$$

X/Y STABILITY CONTOURS FOR +/- SENSITIVITY PAIRS

Plot the Stability Contour Pairs of Log Lg vs Rg/Rdl, Rcs vs Rdl and Rcs vs Log gm

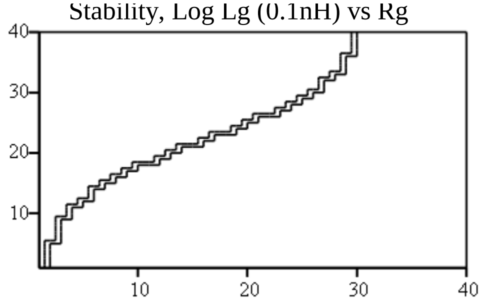
$\text{Lmin} := 1 \quad \text{Lmax} := 10^4 \quad \text{gmin} := 0.1 \quad \text{gmax} := 10 \quad N := 40 \quad i := 1..N \quad j := 1..N \quad RG_i := i \quad RD_i := i \cdot 0.5$

$r_l := \ln\left(\frac{\text{Lmax}}{\text{Lmin}}\right) \qquad LG_j := \text{Lmin} \cdot e^{j \cdot \frac{r_l}{N}} \qquad r_{gm} := \ln\left(\frac{\text{gmax}}{\text{gmin}}\right) \qquad GM_i := \text{gmin} \cdot e^{i \cdot \frac{r_{gm}}{N}}$

Stable Region is at bottom and Rg > 30 Ω

$D_{i,j} := \text{if}\left(\text{DRL}\left(RG_i, \text{Rdl}, LG_j \cdot n, \text{Rcs}, \text{Lcs}, \text{gm}\right) > 0, 1, -1\right)$

Stability, Log Lg (0.1nH) vs Rg

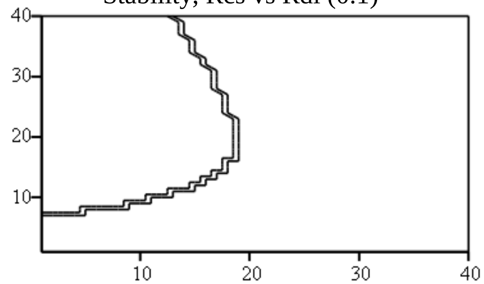


D

Stable Region is at bottom and right

$DSD_{i,j} := \text{if}(\text{DRL}(\text{Rg}, i \cdot 0.1, \text{Lg}, j, \text{Lcs}, \text{gm}) > 0, 1, -1)$

Stability, Rcs vs Rdl (0.1)



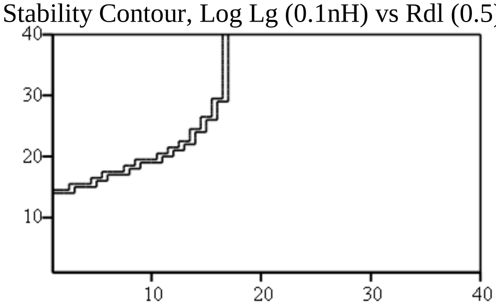
DSD

Stable Region is at bottom and Rdl > 8 Ω

$DD_{i,j} := \text{if}\left(\text{DRL}\left(\text{Rg}, 0.5 \cdot i, LG_j \cdot n, \text{Rcs}, \text{Lcs}, \text{gm}\right) > 0, 1, -1\right)$

$DD_{10,20} = -1 \qquad DD_{2,2} = 1$

Stability Contour, Log Lg (0.1nH) vs Rdl (0.5)



DD

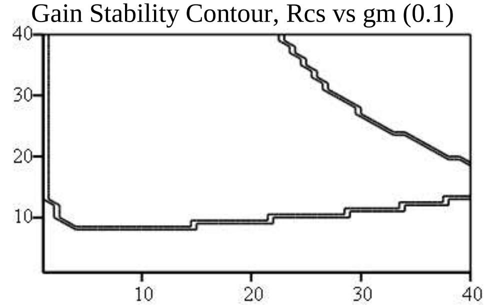
UnStable Region is at bottom left. Lg is 10X

UnStable Region center. USR shrinks w Rg

$DG_{i,j} := \text{if}(\text{DRL}(\text{Rg}, \text{Rdl}, \text{Lg}, j, \text{Lcs}, i \cdot 0.1) > 0, 1, -1)$

$DG_{10,20} = -1 \quad DG_{10,5} = 1 \quad DG_{35,30} = 1$

Gain Stability Contour, Rcs vs gm (0.1)



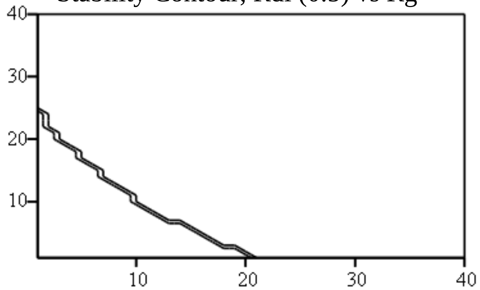
DG

UnStable Region center. USR shrinks w Rg

$GD_{i,j} := \text{if}(DRL(i, 0.5 \cdot j, Lg \cdot 10, Rcs, Lcs, gm) > 0, 1, -1)$

$GD_{10,20} = 1 \quad GD_{2,2} = -1$

Stability Contour, Rdl (0.5) vs Rg

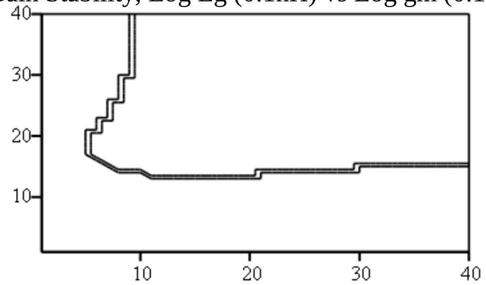


GD

$DL_{i,j} := \text{if}\left(DRL\left(Rg, Rdl, LG_j \cdot n, Rcs, Lcs, GM_i\right) > 0, 1, -1\right)$

$DL_{10,20} = -1 \quad DL_{10,5} = 1$

Gain Stability, Log Lg (0.1nH) vs Log gm (0.1)



DL

DIRECTION OF INSTABILITY CHANGES WITH CAPACITANCE

Below we see that changing the capacitances **singlely, eg. only Cgs,** changes the sign of sensitivities.

$x := 0.1, 0.6 .. 10$ **Vary Cgs with factor x:** $C_{gs}(x) := x \cdot Cgs$

We find that with the exception of Rg and Rdl, which always damp, the sign of d3 and the **signs of all the other sensitivities** flip with decreased Cgs and also Cds. For decreased Cgs or Cds, increasing these factors increases stability. The Cgd capacitances has the opposite effect. Also the effects of the C(s) on the Sensitivities of Lcs, Rg, Rdl, gm are very similar and differ only in magnitude.

List in order of above relative rankings.
For a common plot, multiply by factor (1/|sensirvity|) to scale plot close to 1.

Increasing the parameters with sensitivities in the top half of the plot damps oscillations.
Increasing Rg or Rdl moves all the curves up.

Sign changes (Not Rg/Rdl) Sens'ty vs Cgs

$d3(C_{gs}(x), Cgd, Cds, Rg, Lg, Rdl, Ldl, Rcs, Lcs, gm) \cdot 10^{47.5}$

$\frac{d}{dCgd} d3(C_{gs}(x), Cgd, Cds, Rg, Lg, Rdl, Ldl, Rcs, Lcs, gm) \cdot 10^{37}$

$\frac{d}{dCds} d3(C_{gs}(x), Cgd, Cds, Rg, Lg, Rdl, Ldl, Rcs, Lcs, gm) \cdot 10^{38}$

$\frac{d}{dLg} d3(C_{gs}(x), Cgd, Cds, Rg, Lg, Rdl, Ldl, Rcs, Lcs, gm) \cdot 10^{39}$

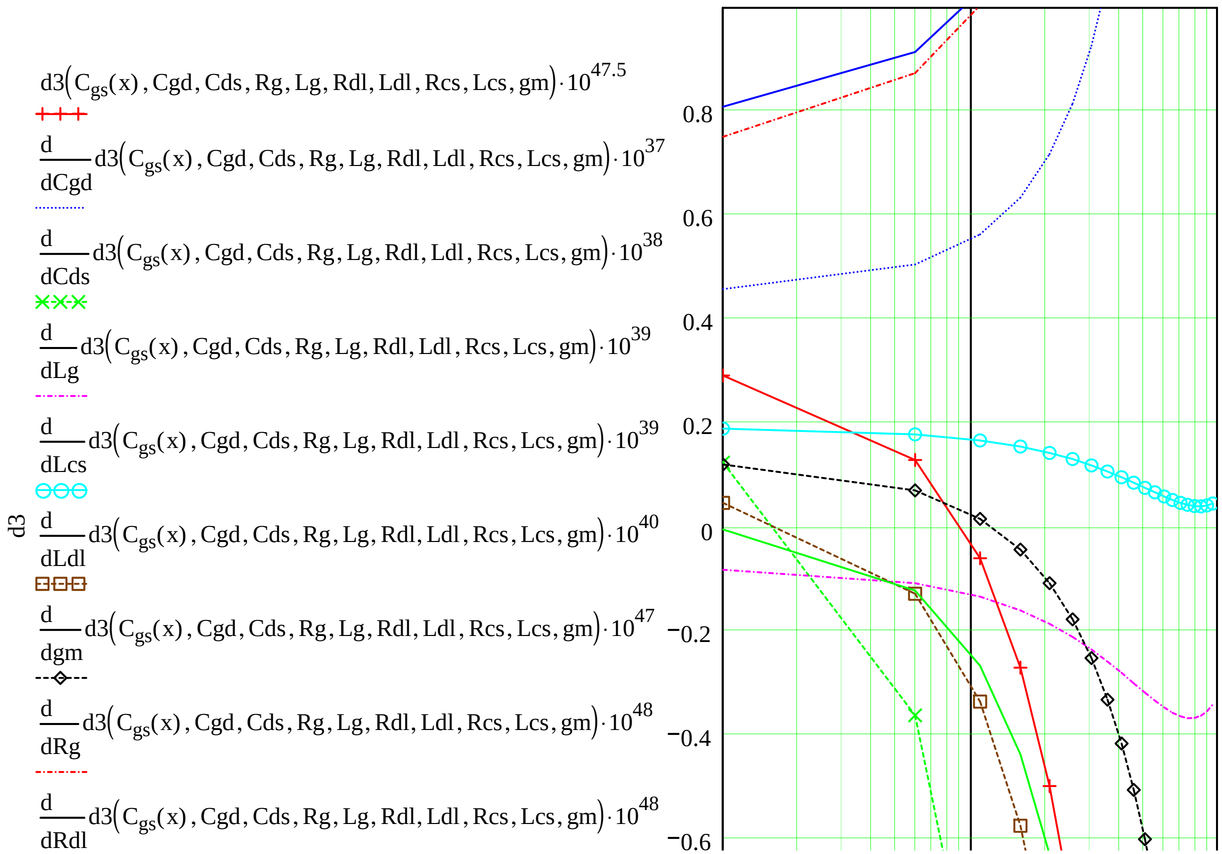
$\frac{d}{dLcs} d3(C_{gs}(x), Cgd, Cds, Rg, Lg, Rdl, Ldl, Rcs, Lcs, gm) \cdot 10^{39}$

$\frac{d}{dLdl} d3(C_{gs}(x), Cgd, Cds, Rg, Lg, Rdl, Ldl, Rcs, Lcs, gm) \cdot 10^{40}$

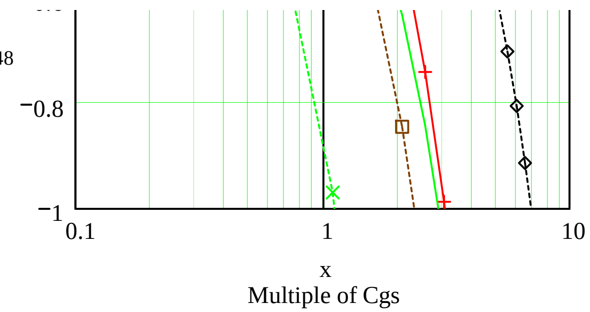
$\frac{d}{dgm} d3(C_{gs}(x), Cgd, Cds, Rg, Lg, Rdl, Ldl, Rcs, Lcs, gm) \cdot 10^{47}$

$\frac{d}{dRg} d3(C_{gs}(x), Cgd, Cds, Rg, Lg, Rdl, Ldl, Rcs, Lcs, gm) \cdot 10^{48}$

$\frac{d}{dRdl} d3(C_{gs}(x), Cgd, Cds, Rg, Lg, Rdl, Ldl, Rcs, Lcs, gm) \cdot 10^{48}$



$\frac{d}{dRcs} d3\left(C_{gs}(x), Cgd, Cds, Rg, Lg, Rdl, Ldl, Rcs, Lcs, gm\right) \cdot 10^{48}$

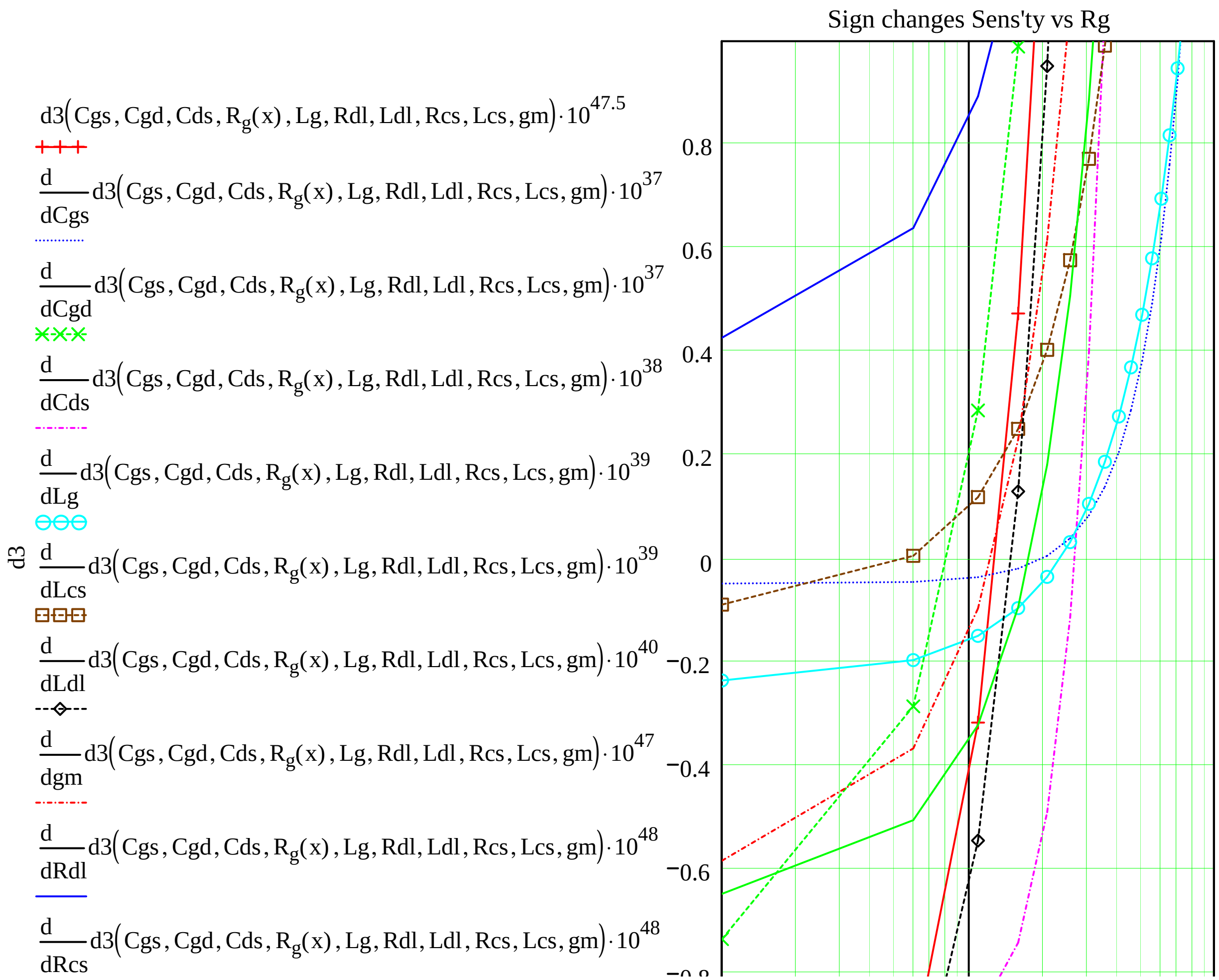


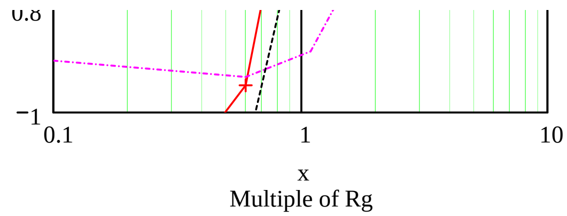
DIRECTION OF INSTABILITY CHANGES WITH Rg

Vary Rg by a factor x:

$x := 0.1, 0.6 .. 10 \qquad R_g(x) := x \cdot 5$

Except for Rdl, the sign of All of the sensitivites flip with Rg and similarly with Rdl.

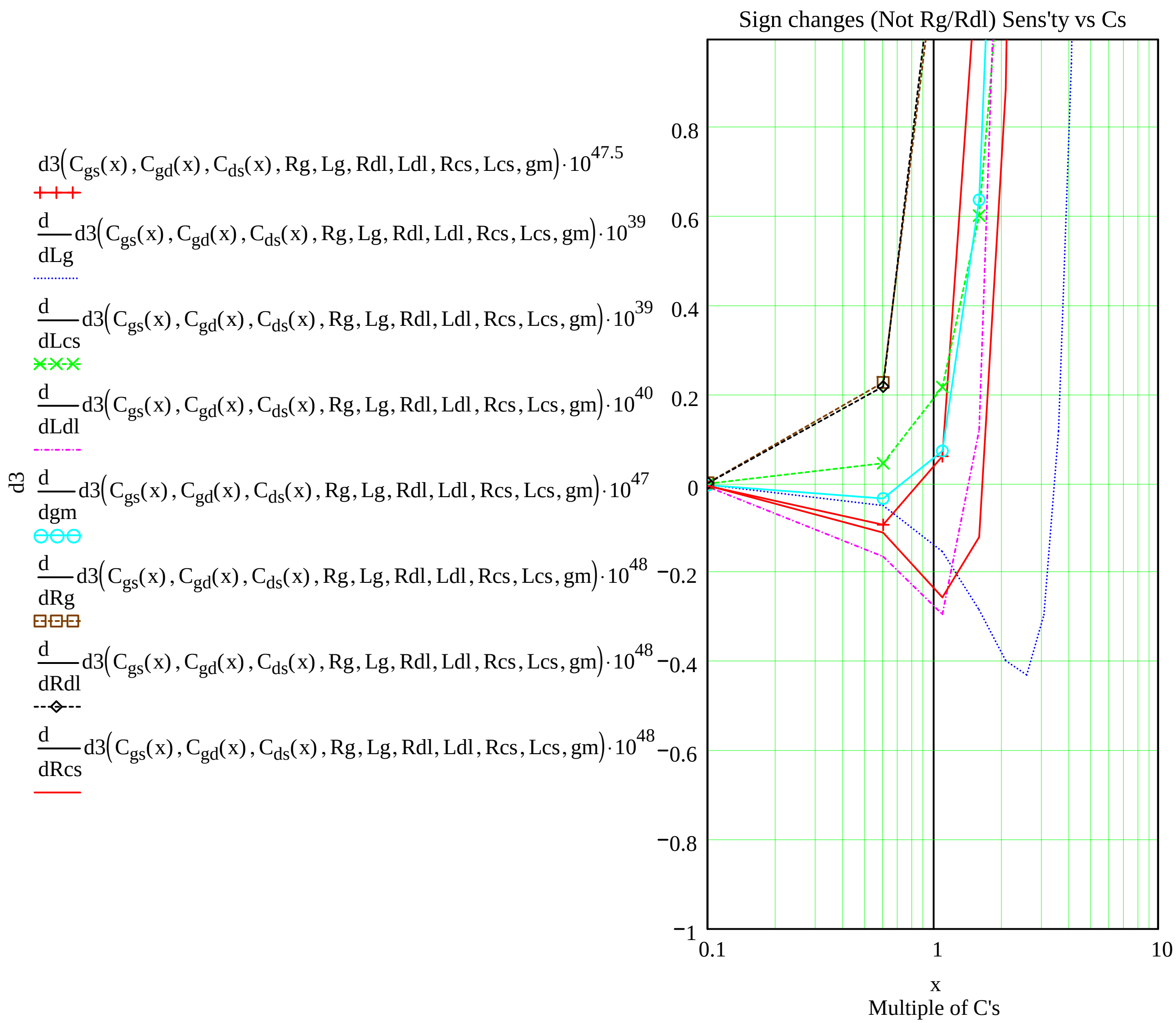



0.8
−1
0.1
1
10
x
Multiple of Rg

CHANGE OF MAGNITUDE OF SENSITIVITY vs. CAPACITANCES

Observe the effect of changing all three of the capacitances **collectively**. Only the magnitude changes.

Vary Cgs, Cgd, Cds with factor x: $C_{gs}(x) := x \cdot Cgs$ $C_{gd}(x) := x \cdot Cgd$ $C_{ds}(x) := x \cdot Cds$



PLOT LOCII OF INSTABILITY

CREATE THE STABILITY FUNCTION, US

Find the Gate Inductance, Lg, at the transition to UnStable Operation for Rg from 1 to 20 ohm and for Cgs 1 to 3 x Cgs for given values of Rcs & Lcs.

```
UG := | for  L ∈ 1..4
      |   | F ← 1
      |   | for  rg ∈ 1..70
      |   |   | d ← 0 + 0.1
      |   |   | Lgg ← 10^L
      |   |   | rgg ← rg + 0.01
      |   |   | D3 ← d3(Cgs, Cgd, Cds, rgg, Lgg·n, d, Ldl, Rcs, Lcs, gm)
      |   |   | while  0 > 10^30·D3
      |   |   |   | D3 ← d3(Cgs, Cgd, Cds, rgg, Lgg·n, d, Ldl, Rcs, Lcs, gm)
      |   |   |   | break  if  (d > 100)
      |   |   |   | d ← d + 1
      |   |   | S_(rg,L) ← d
      |   |   | if  (F = 1)·d = 0.1
      |   |   |   | S_(71,L) ← rg
      |   |   |   | F ← 0
      | S
```

Parameter Approximations for Rdl and Rg for Stable Operation

$$RdlUS(Rg, Lg) := 1 + 6.1 \cdot \log(Lg) - \frac{Rg \cdot 1.6}{\log(Lg)}$$

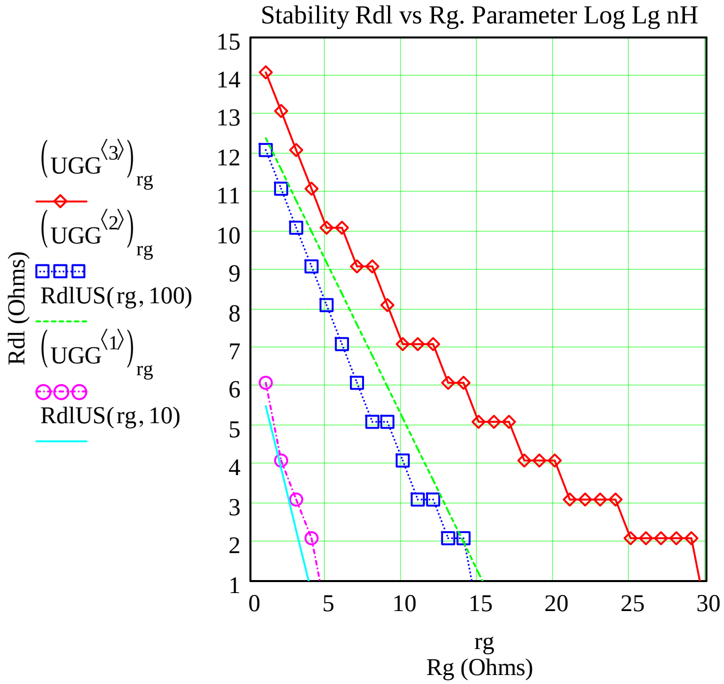
$$RgUS(Rdl, Lg) := \frac{(1 + 6.1 \cdot \log(Lg) - Rdl) \cdot \log(Lg)}{1.6}$$

UGG := UG   rg := 1 .. 70

$d3(Cgs, Cgd, Cds, 2, 10\cdot n, 6, Ldl, Rcs, Lcs, gm) = 9.064 \times 10^{-49}$

$d3(Cgs, Cgd, Cds, 2, 10\cdot n, 2, Ldl, Rcs, Lcs, gm) = -2.821 \times 10^{-49}$

Stable Region is at top. Lg decreases Region of Stability



nn := 1 .. 4 $Rslope_{nn} := \frac{\left(UGG^{\langle nn \rangle}\right)_1}{\left(UGG^{\langle nn \rangle}\right)_{71}} \cdot nn$ $Rslope = \begin{pmatrix} 1.22 \\ 1.613 \\ 1.41 \\ 1.567 \end{pmatrix}$

$nH \equiv 10^{-9}\cdot H$ $nsec \equiv 10^{-9}\cdot sec$ $sq \equiv 1$ $nC \equiv 10^{-9}\cdot C$

Does increasing Rg ever increase instability?

Given $\left[\left(\frac{d}{dRg}d3(Cgs, Cgd, Cds, Rg, Lg, Rdl, Ldl, Rcs, Lcs, gm)\right)\cdot 10^{50}\right] < -1$

$Cgs > 10^{-10}$ $Cgd > 10^{-10}$ $Cds > 10^{-10}$ $Lg > 10^{-8}$ $Rcs > 0$ $Lcs > 10^{-8}$ $gm > 0.1$

$AA := Find(Cgs, Cgd, Cds, Lg, Lcs, Rcs, gm)$

$AA^T = \left(-1.389\times 10^{-9}\ \ 10\times 10^{-11}\ \ 2.14\times 10^{-10}\ \ 3\times 10^{-8}\ \ 1.75\times 10^{-8}\ \ 10\ \ 2\right)$

$\left(\frac{d}{dRg}d3\left(AA_1, AA_2, AA_3, Rg, AA_4, Rdl, Ldl, AA_5, AA_6, AA_7\right)\right) = 5.166\times 10^{-24}$

$$\frac{gs\cdot Lg\cdot Lcs}{}\cdot\lambda^{4} + \frac{Lcs\cdot Rg\cdot Cgd\cdot Cgs + Rdl\cdot Lcs\cdot Cgd\cdot Cgs + Lg\cdot Rcs\cdot Cgd\cdot Cgs + Ldl\cdot Rg\cdot Cgd\cdot Cgs + Lg\cdot Rdl\cdot Cgd\cdot Cgs + Lg\cdot Ldl\cdot gm\cdot Cgd + Lg\cdot Lcs\cdot gm\cdot Cgd + Cds\cdot Rdl\cdot Lcs\cdot Cgd + Cds\cdot Cgs\cdot Rcs\cdot L}{}$$

gs·Ldl·Rg·Cgd + Cgs·Lcs·Rdl·Cgd + Cgs·Cds·Rdl·Lg + Cgs·Lg·Rcs·Cgd + Cgs·Cds·Ldl·Rg + Lg·Cds·Rcs·Cgd + Rg·Lcs·Cgd·Cds + Lcs·gm·Lg·Cgd + Cgs·Rdl·Lg·Cgd + Cgs·Cds·Lg·Rc

Ldl·Cgd + Rg·Lcs·gm·Cgd + Cgs·Cds·Rdl·Rcs + Lcs·Rdl·gm·Cgd + Rdl·Cds·Rcs·Cgd + Rg·Ldl·gm·Cgd + Ldl·Rcs·Cgd·gm + Lg·Cgd + Rcs·Cds·Rg·Cgd + Lg·Rcs·Cgd·gm)·sec^{-2}

$$Cgd + Cgs\cdot Cds\cdot Rg\cdot Lcs + Ldl\cdot Cds\cdot Rcs\cdot Cgd + Cgs\cdot Ldl\cdot Rg\cdot Cgd + Cgs\cdot Lcs\cdot Rdl\cdot Cgd + Cgs\cdot Cds\cdot Rdl\cdot Lg + Cgs\cdot Lg\cdot Rcs\cdot Cgd + Cgs\cdot Cds\cdot Ldl\cdot Rg + Lg\cdot Cds\cdot Rcs\cdot Cgd + Rg\cdot Lcs\cdot Cgd\cdot Cds + Lcs\cdot g$$

$$Rg\cdot Cgd + Rdl\cdot Cds\cdot Rg\cdot Cgd + Rdl\cdot Lg\cdot gm\cdot Cgd + Ldl\cdot Cgd + Rg\cdot Lcs\cdot gm\cdot Cgd + Cgs\cdot Cds\cdot Rdl\cdot Rcs + Lcs\cdot Rdl\cdot gm\cdot Cgd + Rdl\cdot Cds\cdot Rcs\cdot Cgd + Rg\cdot Ldl\cdot gm\cdot Cgd + Ldl\cdot Rcs\cdot Cgd\cdot gm + Lg\cdot Cgd +$$

$$, Lcs, gm) \ ...$$

$$s, Lcs, gm)^2$$

$Lg, Ldl, Lcs))^T$

$$\frac{\cdots dl + Ldl \cdot Lcs \cdot gm \cdot Cgd + Rcs \cdot Ldl \cdot Cgd \cdot Cgs + Cds \cdot Cgs \cdot Lg \cdot Rcs + Cds \cdot Lg \cdot Rdl \cdot Cgd + Cds \cdot Ldl \cdot Rg \cdot Cgd + Cds \cdot Cgs \cdot Rdl \cdot Lcs + Cds \cdot Cgs \cdot Lg \cdot Rdl + Cds \cdot Lg \cdot Rcs \cdot Cgd + Cds \cdot Lcs \cdot Rg \cdot Cgd + Cds \cdot Cg\cdots}{(Lcs \cdot Ldl + Lg \cdot Ldl + Lg \cdot Lcs) \cdot (Cgs \cdot Cgd + Cds \cdot Cgs + Cds \cdot Cgd)}$$

$s + Cgs \cdot Rg \cdot Lcs \cdot Cgd + Ldl \cdot Cds \cdot Rg \cdot Cgd) \cdot sec^{-3}$

;m·Lg·Cgd + Cgs·Rdl·Lg·Cgd + Cgs·Cds·Lg·Rcs + Cgs·Rg·Lcs·Cgd + Ldl·Cds·Rg·Cgd)

Rcs·Cds·Rg·Cgd + Lg·Rcs·Cgd·gm)

$$\frac{s\cdot Lcs\cdot Rg + Cds\cdot Rcs\cdot Ldl\cdot Cgd + Cds\cdot Cgs\cdot Ldl\cdot Rg}{}\cdot\lambda^{3} + \frac{Cds\cdot Rcs\cdot Rg\cdot Cgd + Cgs\cdot Lg + Cds\cdot Lcs + Cds\cdot Ldl + Cds\cdot Cgs\cdot Rdl\cdot Rg + Lg\cdot Cgd + Ldl\cdot Cgd + Rdl\cdot Rg\cdot Cgd\cdot Cgs + Cgs\cdot Lcs + Cds\cdot Cgs\cdot}{}$$

$$\frac{Rdl\cdot Rcs + Rcs\cdot Rg\cdot Cgd\cdot Cgs + Lg\cdot Rcs\cdot gm\cdot Cgd + Rdl\cdot Lcs\cdot gm\cdot Cgd + Lcs\cdot Rg\cdot gm\cdot Cgd + Cds\cdot Cgs\cdot Rcs\cdot Rg + Ldl\cdot Rg\cdot gm\cdot Cgd + Rcs\cdot Ldl\cdot gm\cdot Cgd + Lg\cdot Rdl\cdot gm\cdot Cgd + Rdl\cdot Cgs\cdot Rcs\cdot Cgd + Cd}{(Lcs\cdot Ldl + Lg\cdot Ldl + Lg\cdot Lcs)\cdot(Cgs\cdot Cgd + Cds\cdot Cgs + Cds\cdot Cgd)}$$

$$\frac{\text{s}\cdot Rdl\cdot Rg\cdot Cgd + Cds\cdot Rdl\cdot Rcs\cdot Cgd}{}\cdot\lambda^{2} + \frac{Rdl\cdot Cgd + Cds\cdot Rdl + Rcs\cdot Cgs + Rdl\cdot Rg\cdot gm\cdot Cgd + Rg\cdot Cgd + Rdl\cdot Rcs\cdot gm\cdot Cgd + Rcs\cdot Rg\cdot gm\cdot Cgd + Lcs\cdot gm + Rcs\cdot Cds + Rg\cdot Cgs}{(Lcs\cdot Ldl + Lg\cdot Ldl + Lg\cdot Lcs)\cdot(Cgs\cdot Cgd + Cds\cdot Cgs + Cds\cdot Cgd)}\cdot\lambda + \frac{}{(Lcs\cdot Ldl}$$

$$\frac{1 + Rcs \cdot gm}{+ Lg \cdot Ldl + Lg \cdot Lcs) \cdot (Cgs \cdot Cgd + Cds \cdot Cgs + Cds \cdot Cgd)}$$