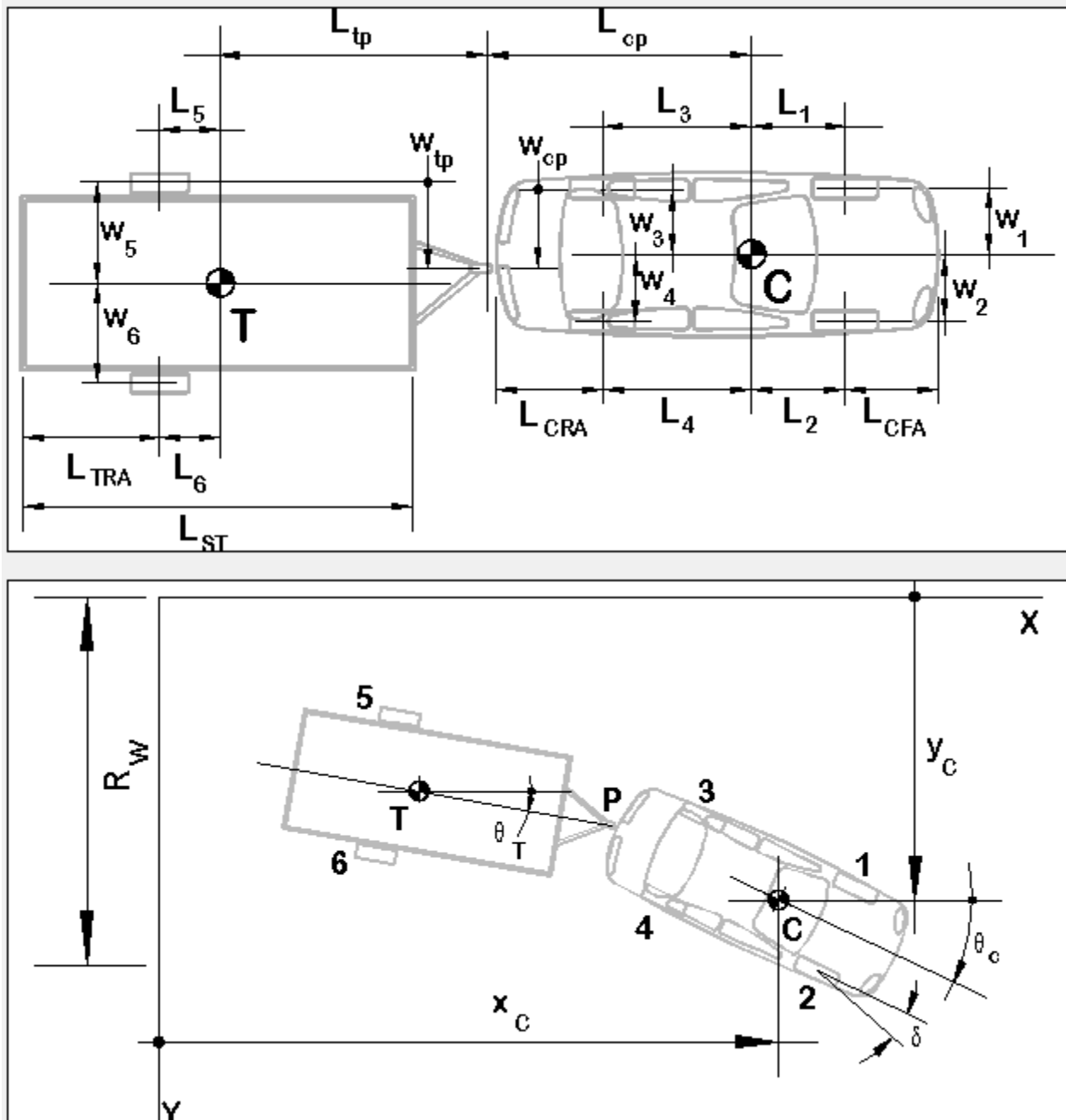


Articulated Vehicle Planar Simulation

Simulate the Planar Motion of an articulated vehicle in time

YXPhysics.com

Layout of Physical Variables



Brach & Brach *Vehicle Accident Analysis and Reconstruction Methods* - Fig.13.1

Articulated Vehicle Planar Simulation

Input Variables

For Definitions of Variables Refer to Brach & Brach *Vehicle Accident Analysis and Reconstruction Methods*.
 13.5 Appendix: Differential Equations of Planar Vehicle Motion

$$\begin{pmatrix} W_c & J_c \\ L_1 & L_2 \\ L_3 & L_4 \\ W_1 & W_2 \\ W_3 & W_4 \\ L_{cp} & W_{cp} \\ h_c & h_{cx} \\ C_{a1} & C_{a2} \\ C_{a3} & C_{a4} \\ C_{s1} & C_{s2} \\ C_{s3} & C_{s4} \\ s_1 & s_2 \\ s_3 & s_4 \\ C_{t1} & C_{t2} \\ C_{t3} & C_{t4} \\ X_{c0} & V_{xc0} \\ Y_{c0} & V_{yc0} \\ \theta_{c0} & \theta_{vc0} \\ \delta & g \end{pmatrix} = \begin{pmatrix} 3000 & 1900 & W.c & J.c \\ 3.75 & 3.75 & L.1 & L.2 \\ 4.58 & 4.58 & L.3 & L.4 \\ 2.5 & 2.5 & W.1 & W.2 \\ 2.5 & 2.5 & W.3 & W.4 \\ 3 & 0 & L.cp & W.cp \\ 1 & 0 & h.c & h.cx \\ 8800 & 8800 & C.a1 & C.a2 \\ 8100 & 8100 & C.a3 & C.a4 \\ 10000 & 10000 & C.s1 & C.s2 \\ 10000 & 10000 & C.s3 & C.s4 \\ 0 & 0 & s.1 & s.2 \\ 0.007 & 0.007 & s.3 & s.4 \\ 0 & 0 & C.t1 & C.t2 \\ 0 & 0 & C.t3 & C.t4 \\ 0 & 73.3 & X.c0 & V.xc0 \\ 0 & 0 & Y.c0 & V.yc0 \\ 0 & 0 & \theta.c0 & \theta.vc0 \\ 0 & 32.17 & \delta & g \end{pmatrix}$$

$$\begin{pmatrix} W_t & J_t \\ L_3 & L_4 \\ W_5 & W_6 \\ L_{tp} & W_{tp} \\ h_t & h_{tx} \\ C_{a5} & C_{a6} \\ C_{s5} & C_{s6} \\ s_5 & s_6 \\ C_{t5} & C_{t6} \\ \theta_{t0} & \theta_{vt0} \end{pmatrix} = \begin{pmatrix} 1900 & 0 \\ 2 & 2 \\ 3.7 & 3.7 \\ 6.5 & 3.7 \\ 0.5 & 0 \\ 9200 & 9200 \\ 10000 & 10000 \\ 0 & 0 \\ 0 & 0 \\ 0 & 40 \end{pmatrix} \begin{pmatrix} W.t & J.t \\ L.3 & L.4 \\ W.5 & W.6 \\ L.tp & W.tp \\ h.t & h.tx \\ C.a5 & C.a6 \\ C.s5 & C.s6 \\ s.5 & s.6 \\ C.t5 & C.t6 \\ \theta.t0 & \theta.vt0 \end{pmatrix}$$

$$m_c := \frac{W_c}{g} \quad m_t := \frac{W_t}{g} \quad F_{cx} := 0 \quad F_{xy} := 0 \quad f := 0.7 \quad F_{xx} := \sum_{n=1}^6 f_{ix}$$

$$m_{bar} := \frac{m_c \cdot m_t}{m_c + m_t} \quad m_{ct} := m_c + m_t \quad F_{cx} := f \cdot m_c \quad F_{cy} := 0$$

$$F_{ty} := 0 \quad F_{tx} := f \cdot m_t$$

$$t_e := 2 \quad FRAME := 100$$

$$t_1 := \frac{t_e}{100} \cdot (FRAME + 1) \quad t_1 = 2.02$$

Derivative Symbol Ctrl+F7

Differential Equations of Motion of Vehicles

$$R_{cp} := \sqrt{L_{cp}^2 + (W_3 - W_{cp})^2}$$

$$\theta_{cp}(\theta) := \frac{\pi}{2} - \theta - \operatorname{atan}\left[\frac{(W_3 - W_{cp})}{L_{cp}}\right]$$

$$r_{tpx}(\theta) := -L_{tp} \cdot \sin(\theta) - (W_{tp} - W_5) \cos(\theta)$$

$$r_{cpx}(\theta) := L_{cp} \cdot \sin(\theta) + (W_3 - W_{cp}) \cos(\theta)$$

$$r_{cpy}(\theta) := -L_{cp} \cdot \cos(\theta) + (W_3 - W_{cp}) \sin(\theta)$$

$$R_{tp} := \sqrt{L_{tp}^2 + (W_{tp} - W_5)^2}$$

$$\theta_{tp}(\theta) := \frac{\pi}{2} - \theta - \operatorname{atan}\left[\frac{(W_{tp} - W_5)}{L_{tp}}\right]$$

$$r_{tpy}(\theta) := L_{tp} \cdot \cos(\theta) - (W_{tp} - W_5) \sin(\theta)$$

$$r_{tpx}(\theta) := -L_{cp} \cdot \sin(\theta) - (W_{tp} - W_5) \cos(\theta)$$

Given

$$Xc(0) = X_{c0} \quad Xc'(0) = V_{xc0} \quad Yc(0) = Y_{c0} \quad Yc'(0) = V_{yc0} \quad \theta c(0) = \theta_{c0} \quad \theta c'(0) = 0 \quad \theta t(0) = \theta t'(0) = 0$$

$$m_{ct} \cdot Xc''(t) = -m_t \left[-R_{cp} \cdot \theta c''(t) \cos(\theta_{cp}(\theta c(t))) + R_{cp} \cdot (-2 \theta_{cp}'(\theta c(t))) \theta c'(t) \sin(\theta_{cp}(\theta c(t))) + R_{tp} \cdot \theta t''(t) \cos(\theta_{tp}(\theta t(t))) \right] \dots \\ + m_t \cdot R_{tp} \cdot 2 \theta t(t) \theta t'(t) \sin(\theta_{tp}(\theta t(t))) + F_{cx} + F_{tx}$$

$$m_{ct} \cdot Yc''(t) = -m_t \left(-R_{cp} \cdot \theta c''(t) \sin(\theta_{cp}(\theta c(t))) + R_{cp} \cdot 2 \theta c(t) \theta c'(t) \cos(\theta_{cp}(\theta c(t))) - R_{tp} \cdot \theta t''(t) \sin(\theta_{tp}(\theta t(t))) \right) \dots \\ + m_t \cdot R_{tp} \cdot 2 \theta t(t) \theta t'(t) \cos(\theta_{tp}(\theta t(t))) + F_{cy} + F_{ty}$$

$$\left[J_c + m_{bar} \cdot R_{cp} \cdot (r_{cpx}(\theta c(t)) \cdot \cos(\theta_{cp}(\theta c(t))) - r_{cpy}(\theta c(t)) \cdot \sin(\theta_{cp}(\theta c(t)))) \right] \theta c''(t) = f \cdot m_{bar}$$

$$\left[J_t + m_{bar} \cdot R_{tp} \cdot (r_{tpy}(\theta t(t)) \cdot \sin(\theta_{tp}(\theta t(t))) - r_{tpx}(\theta t(t)) \cdot \cos(\theta_{tp}(\theta t(t)))) \right] \theta t''(t) = f \cdot m_{bar}$$

$$\begin{pmatrix} Xc \\ Yc \\ \theta c \\ \theta t \end{pmatrix} := \text{Odesolve} \left[\begin{pmatrix} Xc \\ Yc \\ \theta c \\ \theta t \end{pmatrix}, t, t_1 \right]$$

$$n := 0..20$$

$$tt_n := n \cdot 0.1$$

$$V_x(t) := \frac{d}{dt} Xc(t)$$

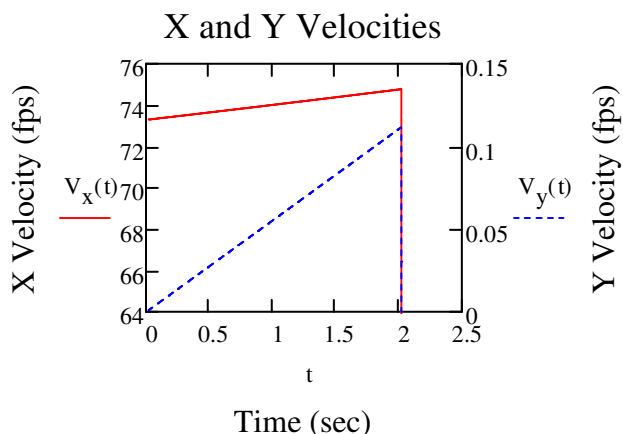
$$V_y(t) := \frac{d}{dt} Yc(t)$$

$$V_x(1) = 74.023$$

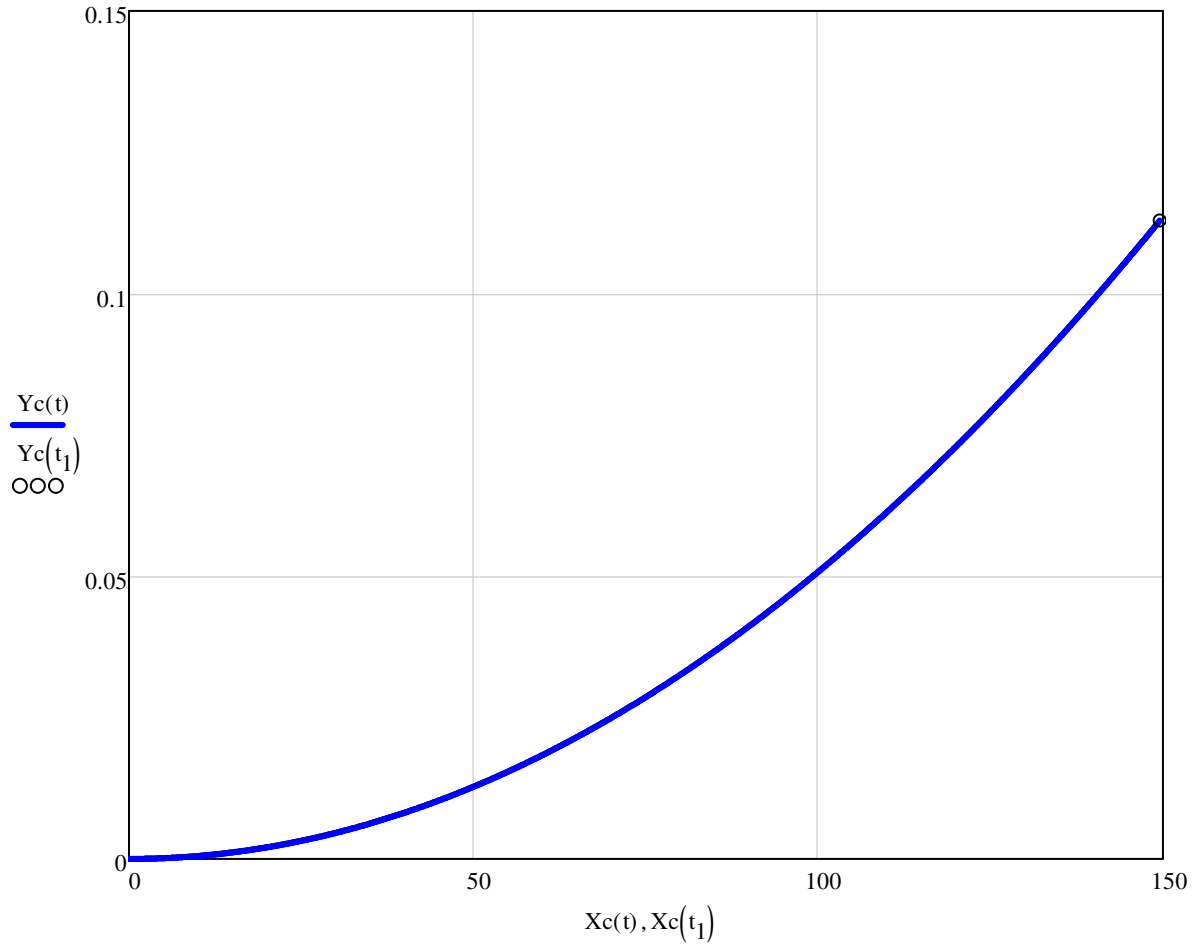
$$X_n := Xc(tt_n) \quad Y_n := Yc(tt_n) \quad V_{yn} := V_y(tt_n)$$

$$\theta_{cn} := \theta c(tt_n) \quad \theta_{tn} := \theta t(tt_n)$$

$$\text{Output} := \text{augment}(tt, X, Y, \theta_c, \theta_t)$$



Vehicle Path



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